

MAC 2312 (Calculus II) — Answers
 QUIZ 4, Friday September 16, 2016

Name:

PID:

Remember to show all your work; you won't get any credits if you do not show the steps to your answers.

1. [3] Use integration by parts to evaluate the integral:

$$\int_1^2 \sqrt{x} \ln x dx = \frac{2}{3} x^{3/2} \ln x \Big|_1^2 - \int_1^2 \frac{2}{3} x^{3/2} \cdot \frac{1}{x} dx = \frac{2}{3} (2\sqrt{2} \ln 2 - \ln 1) - \frac{2}{3} \int_1^2 x^{1/2} dx$$

$$= \frac{4}{3} \sqrt{2} \ln 2 - \frac{4}{9} x^{3/2} \Big|_1^2, \text{ as } \ln 1 = 0$$

$$= \frac{4}{3} \sqrt{2} \ln 2 - \frac{4}{9} (2\sqrt{2} - 1)$$

$u = \ln x$
 $du = \frac{dx}{x}$
 $dv = \sqrt{x} dx = x^{1/2} dx$
 $v = \frac{2}{3} x^{3/2}$

2. [3] Evaluate the integral

$$\int \cos^3 x dx = \int \cos^2 x \cdot \cos x dx = \int (1 - u^2) du = u - \frac{u^3}{3} + C$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$

Set $u = \sin x$
 $du = \cos x dx$

3. [2] Evaluate the integral:

$$\int \sec^2(\tan x) \sec^2(x) dx = \int \sec^2(u) du = \tan(u) + C = \tan(\tan x) + C$$

$u = \tan x$
 $du = \sec^2 x dx$

4 [2] Given that $\ln a = -2$ and $\ln c = 5$, evaluate the integral by using the definition of the natural logarithmic function:

$$\int_a^{c/a^3} \frac{1}{t} dt = \int_a^1 \frac{1}{t} dt + \int_1^{c/a^3} \frac{1}{t} dt,$$

$$= -\ln a + \ln \frac{c}{a^3}, \text{ by definition of } \ln$$

$$= -\ln a + \ln c - \ln(a^3), \text{ by quotient property of } \ln$$

$$= -\ln a + \ln c - 3 \ln a$$

$$= -4 \ln a + \ln c = -4(-2) + 5 = 13$$