

MAC 2313 (Multivariable Calculus) — Answers  
 QUIZ 4, Friday September 16, 2016

Name:

PID:

Remember that no documents or calculators, or any other electronic devices are allowed during the quiz. Also remember that you won't get any credit(s) if you do not show the steps to your answers.

1. [3] Describe the level surface  $f(x, y, z) = k$  in words for each value of  $k$  if  $f(x, y, z) = y^2 - 4x^2 + 2z^2$ , and  $k = 1, 0, -1$ .

$k = 1$ :  $y^2 - 4x^2 + 2z^2 = 1$ , hyperboloid of one sheet along  $x$ -axis

$k = 0$ :  $\frac{y^2}{4} + \frac{z^2}{2} = x^2$ , elliptic cone along  $x$ -axis

$k = -1$ :  $y^2 - 4x^2 + 2z^2 = -1$  or  $4x^2 - y^2 - 2z^2 = 1$ , hyperboloid of two sheets along  $x$ -axis

2. [2] Consider the surface whose equation is  $z - 3x^2 - 3y^2 = 0$ . a) Identify the surface. b) Write down the equation and identify the surface obtained by reflecting the given surface about the plane i)  $x = z$ , ii)  $y = z$ , iii)  $z = 0$ .

a)  $z = 3x^2 + 3y^2$ ; (circular) paraboloid along  $z$ -axis

b) i) switch  $x$  and  $z$ :  $x = 3z^2 + 3y^2$ , paraboloid along  $x$ -axis

ii) switch  $y$  and  $z$ :  $y = 3x^2 + 3z^2$ , paraboloid along  $y$ -axis

iii) change  $z$  to  $-z$ :  $z = -3x^2 - 3y^2$ , upside down paraboloid along  $z$ -axis

3. [3] The equation of a surface in spherical coordinates is given by:  $\rho = 2 \sin(\phi) \sin(\theta)$ . Convert it to rectangular coordinates, and identify the surface.

Equation may be rewritten

$\rho^2 = 2\rho \sin\phi \sin\theta$ , by multiplying both sides by  $\rho$

In rectangular coordinates, we get

$x^2 + y^2 + z^2 = 2y$  or  $x^2 + (y-1)^2 + z^2 = 1$ ; sphere centered

at  $(0, 1, 0)$  with radius  $r = 1$ .

4. [2] Convert the point  $(-2, -2, 3)$  from rectangular coordinates to cylindrical coordinates.

$r = \sqrt{(-2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$\tan\theta = \frac{-2}{-2} = 1$   
 $\theta$  in 3rd quadrant } so  $\theta = \frac{5\pi}{4}$

In cylindrical coordinates, we get  $(2\sqrt{2}, \frac{5\pi}{4}, 3)$