MAP 2302 (Differential Equations) — Answers
QUIZ 5, Friday February 23, 2018

Name: **PID:**

1. [4] Solve the differential equation: $y''' - 8y = 0$

Aux. Eqn: $m^3 - 8 = 0$ or $(m-2)(m^2+2m+4) = 0$

$m_1 = 2$, $m^2+2m+4 = (m+1)^2 + 3 = 0 \rightarrow (m+1)^2 = -3 = (i\sqrt{3})^2$

$m_2 = -1 + i\sqrt{3}$, $m_3 = -1 - i\sqrt{3}$

$y_c = c_1 e^{2x} + (c_2 \cos(x\sqrt{3}) + c_3 \sin(x\sqrt{3}))$, c_1, c_2, c_3 = Constants

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2. [6] Use the method of undetermined coefficients to solve the differential equation: $y'' - 4y' + 4y = e^{2x}$

Homogeneous D.E:

$y'' - 4y' + 4y = 0$

Aux. Eqn: $m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m_1 = 2 = m_2$

$y_c = (c_1 + c_2 x) e^{2x}$

UC set: $S = \{e^{2x}\}$. Now e^{2x} and xe^{2x} solves the homogeneous D.E

So $S_{new} = \{x^2 e^{2x}\}$. Seek $y_p = Ax^2 e^{2x}$, A = constant to be determined

$y_p' = A(2x + 2x^2)e^{2x}$, $y_p'' = A(2 + 4x + 4x + 4x^2)e^{2x}$

$y_p'' - 4y_p' + 4y_p = [A(2 + 8x + 4x^2) - A(8x + 8x^2) + 4Ax^2]e^{2x}$

$= 2Ae^{2x}$

$= e^{2x} \rightarrow 2A = 1 \rightarrow A = \frac{1}{2}$

Hence $y_p = \frac{x^2}{2} e^{2x}$

General soln of D.E: $y = y_c + y_p$

$= \left(c_1 + c_2 x + \frac{x^2}{2}\right) e^{2x}$, c_1, c_2 = constants