

MAC 2313 (Multivariable Calculus) - Answer
 QUIZ 6, Friday September 30, 2016

Name:

PID:

Remember that no documents or calculators, or any other electronic devices are allowed during the quiz. Also remember that you won't get any credit(s) if you do not show the steps to your answers.

1. [3] Let $f(x, y, z) = \sin(xyz)$. Find the differential df of the function f .

$$\begin{aligned} df(x, y, z) &= \frac{\partial f}{\partial x}(x, y, z)dx + \frac{\partial f}{\partial y}(x, y, z)dy + \frac{\partial f}{\partial z}(x, y, z)dz \\ &= yz \cos(xyz)dx + xz \cos(xyz)dy + xy \cos(xyz)dz \end{aligned}$$

2. [4] Let g be differentiable at the point $P(-1, 1, 3)$, with $g(P) = 7$, $g_x(P) = -2$, $g_y(P) = 4$ and $g_z(P) = -3$.

a) Write down the local linear approximation L for g about P .

$$\begin{aligned} L(x, y, z) &= g(P) + g_x(P)(x+1) + g_y(P)(y-1) + g_z(P)(z-3) \\ &= 7 - 2(x+1) + 4(y-1) - 3(z-3) \quad 0.5 \text{ for each correct term} \end{aligned}$$

b) Use the local linear approximation L to approximate $g(-1.02, 0.97, 2.99)$.

$$\begin{aligned} g(-1.02, 0.97, 2.99) &\approx L(-1.02, 0.97, 2.99) = 7 - 2(-0.02) + 4(-0.03) - 3(-0.01) \\ &\approx 7 + 0.04 - 0.12 + 0.03 \quad 0.5 \quad 1.5 \\ &\approx 6.95 \end{aligned}$$

3. [3] Let f be a differentiable function of two variables (u, v) . Set $z(x, y) = f(u, v)$. If $u = x^2 - 3y$ and $v = 2x^2 + 3y^2$, use an appropriate chain rule to find the partial derivatives $z_x(x, y)$ and $z_y(x, y)$. Your answers may depend on x, y, u and v ; don't worry about that.

$$\begin{aligned} z_x(x, y) &= \frac{\partial}{\partial x} f(u, v) = u_x f_u(u, v) + v_x f_v(u, v) \\ &= 2x f_u(u, v) + 4x f_v(u, v) \quad 0.5 \end{aligned}$$

$$\begin{aligned} z_y(x, y) &= \frac{\partial}{\partial y} f(u, v) = u_y f_u(u, v) + v_y f_v(u, v) \\ &= -3 f_u(u, v) + 6y f_v(u, v) \quad 0.5 \end{aligned}$$