MAC 2312 (Calculus II) — Answers
QUIZ 7, Friday October 14, 2016

Name: **PID:**

Remember to show all your work; you won't get any credits if only your answers are shown without the steps leading to them.

1. [4] Use the trapezoidal rule, then the Simpson's rule, both with $n = 2$ to approximate the integral $\int_1^2 e^{\frac{1}{x^3}}\,dx$.

$x_0 = 1 \quad x_1 = \frac{3}{2} \quad 2 = x_2$

Trapez. Rule: $\int_1^2 e^{\frac{1}{x^3}}dx \approx \frac{1}{4}(e + 2e^{\frac{8}{27}} + e^{\frac{1}{8}}) = T_2$

Simpson's Rule: $\int_1^2 e^{\frac{1}{x^3}}dx \approx \frac{1}{6}(e + 4e^{8/27} + e^{\frac{1}{8}})$

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2. [2] Let $(u_n)_n$ be the sequence given by $u_n = \frac{3^n}{3^n+1}$, $n = 0, 1, 2, \dots$ Use the ratio $\frac{u_{n+1}}{u_n}$ to show that the sequence $(u_n)_n$ is strictly increasing.

$$\frac{u_{n+1}}{u_n} = \frac{3^{n+1}}{3^{n+1}+1} \cdot \frac{3^n+1}{3^n} = \frac{\cancel{3^n} \cdot 3}{\cancel{3^n}} \cdot \frac{(3^n+1)}{3^{n+1}+1}$$

$$= \frac{3(3^n+1)}{3^{n+1}+1}$$

$$= \frac{3^{n+1}+3}{3^{n+1}+1} > 1, \text{ for all } n$$

So $(u_n)_n$ is strictly increasing

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3. [4] Let $(v_n)_n$ be the sequence defined by $v_n = \frac{(-1)^n(2n^2+3)}{4n^2+3}$, $n = 1, 2, \dots$ a) Write down the first four terms of the sequence $(v_n)_n$. b) Decide whether the sequence $(v_n)_n$ converges or diverges; be careful to explain your answer, or no credit.

a) $v_1 = -\frac{5}{7}, \quad v_2 = \frac{11}{19}, \quad v_3 = -\frac{21}{39}, \quad v_4 = \frac{35}{67}$

b) $v_{2n} = \frac{2(2n)^2+3}{4(2n)^2+3} = \frac{8n^2+3}{16n^2+3}, \quad n = 1, 2, \dots$

$$\lim_{n\to\infty} v_{2n} = \lim_{n\to\infty} \frac{\cancel{n^2}(8 + \frac{3}{n^2})}{\cancel{n^2}(16 + \frac{3}{n^2})} = \frac{8}{16} = \frac{1}{2}$$

$v_{2n+1} = -\frac{2(2n+1)^2+3}{4(2n+1)^2+3}, \quad n = 0, 1, 2, \dots$

$$\lim_{n\to\infty} v_{2n+1} = -\lim_{n\to\infty} \frac{\cancel{n^2}(2(2+\frac{1}{n})^2 + \frac{3}{n^2})}{\cancel{n^2}(4(2+\frac{1}{n})^2 + \frac{3}{n^2})} = -\frac{2(4)}{4(4)} = -\frac{1}{2} \neq \lim_{n\to\infty} v_{2n}$$

So (v_n) diverges