MAC 2313 (Multivariable Calculus) – Answers
QUIZ 8, Friday October 21, 2016

Name: **PID:**

Remember that no documents or calculators, or any other electronic devices are allowed during the quiz. Also remember that you won't get any credit(s) if you do not show the steps to your answers.

1. [5] Let $f(x, y) = 3x - 4y$. Use Lagrange multipliers to find the absolute maximum and minimum values of f on the circle $x^2 + y^2 = 4$.

Set $h(x,y) = 3x - 4y - \lambda(x^2+y^2-4)$

$\nabla h(x,y) = \langle 3-2\lambda x, -4-2\lambda y\rangle$ 0.5 (each component)

$\nabla h(x,y) = \vec{0} \to 3-2\lambda x = 0 \to x = \frac{3}{2\lambda}$; $2+\lambda y = 0 \to y = -\frac{2}{\lambda}$ 0.5

$x^2+y^2 = 4 \to \frac{9}{4\lambda^2} + \frac{4}{\lambda^2} = 4$ 0.5 or $9+16 = 16\lambda^2$

0.5 $\lambda^2 = \frac{25}{16}$. So $\lambda = \pm\frac{5}{4}$ 0.5

$\lambda = \frac{5}{4} \to x = \frac{6}{5}$ 0.25, $y = -\frac{8}{5}$ 0.25; $f(\frac{6}{5}, -\frac{8}{5}) = \frac{18}{5} + \frac{32}{5} = \frac{50}{5} = 10$ 0.25 ← absolute maximum 0.25

$\lambda = -\frac{5}{4} \to x = -\frac{6}{5}$ 0.25, $y = \frac{8}{5}$ 0.25; $f(-\frac{6}{5}, \frac{8}{5}) = -\frac{18}{5} - \frac{32}{5} = -\frac{50}{5} = -10$ 0.25 ← absolute minimum 0.25

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2. [5] Find and classify all the critical points of $f(x, y) = x^3 + 3xy - y^2 - 2$ as points of local minimum, local maximum, or saddle point.

$f_x(x,y) = 3x^2 + 3y$, $f_y(x,y) = 3x - 2y$ ← 0.25 for each derivative

$f_x(x,y) = 0 \to x^2+y = 0 \to y = -x^2$ 0.25

$f_y(x,y) = 0 \to 3x-2y = 0 \to y = \frac{3x}{2}$ 0.25

So $\frac{3x}{2} = -x^2$ or $\frac{3x}{2} + x^2 = 0$

or $x(\frac{3}{2}+x) = 0$ 0.25 $\to x = 0$ 0.25 or $x = -\frac{3}{2}$ 0.25

$x = 0 \to y = 0$ 0.25, $x = -\frac{3}{2} \to y = -\frac{9}{4}$ 0.25

CPs: $(0,0)$, $(-\frac{3}{2}, -\frac{9}{4})$ 0.25 each

$f_{xx}(x,y) = 6x$, $f_{xy} = 3$, $f_{yy} = -2$

0.5 $\Delta(x,y) = 9 + 12x = f_{xy}(x,y)^2 - f_{xx}(x,y)f_{yy}(x,y)$

0.25 $\Delta(0,0) = 9 > 0$: f has a saddle point at $(0,0)$ 0.25

$\Delta(-\frac{3}{2}, -\frac{9}{4}) = 9-18 = -9 < 0$ 0.25, $f_{xx}(-\frac{3}{2}, -\frac{9}{4}) = -9 < 0$ 0.25; f has a local maximum at 0.25 $(-\frac{3}{2}, -\frac{9}{4})$