Thin elastic plates involving fractional rotational forces: semigroups regularity and stability

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Overview

- Thermoelastic plate with fractional rotational forces

Collaborators: Valentin Keyantuo (University of Puerto Rico, San Juan) and Mahamadi Warma (George Mason University)

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- Thermoelastic plate with fractional rotational forces
  - Model formulation

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- Some comments

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Model formulation

Let $\theta \in [0,1]$. Consider the thermoelastic plate equations

$$
\begin{aligned}
&B_\theta y_{tt} + \Delta^2 y + \alpha\Delta z = 0 \text{ in } \Omega \times (0,\infty)\\
&z_t - \kappa\Delta z - \beta\Delta y_t = 0 \text{ in } \Omega \times (0,\infty)\\
&y(0) = y^0 \in V, \quad y_t(0) = y^1 \in H_\theta = D(B_\theta^{\frac{1}{2}}), \quad z(0) = z^0 \in H,
\end{aligned}
$$

where $B_\theta u = u + (-\Delta)^\theta u$, with the boundary conditions:
(Hinged plate/Dirichlet temperature):

$$z = 0, \quad y = 0, \quad \Delta y = 0 \text{ on } \Sigma = \partial\Omega \times (0,\infty),$$

or, else
(Clamped plate/Dirichlet temperature):

$$z = 0, \quad y = 0, \quad \partial_\nu y = 0 \text{ on } \Sigma = \partial\Omega \times (0,\infty).$$

Brief literature

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- Lasiecka-Triggiani (1998): proved lack of analyticity results for the semigroup associated with a Kirchhoff thermoelastic plate with various boundary conditions.

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Gevrey regularity

Question: Given that the semigroup associated with the Euler-Bernoulli thermoelastic plate ($\theta = 0$) is analytic, and the one associated with the Kirchhoff thermoelastic plate ($\theta = 1$) is not analytic, what can be said about the semigroup associated with the intermediate model $0 < \theta < 1$?

Gevrey regularity

$$V_1 = H^2(\Omega) \cap H_0^1(\Omega), \quad H_\theta = H^\theta(\Omega) = D(B_\theta^{\frac{1}{2}}),$$
$$W = H_0^1(\Omega), \quad V_2 = H_0^2(\Omega), \quad H = L^2(\Omega)$$

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For $j = 1,\ 2$, we introduce the Hilbert space $\mathcal{H}_{\theta,j} = V_j \times H_\theta \times H$ over the field of complex numbers, and we endow it with the norm given by (notice that the norm is the same in both cases):

$$||(u, v, w)||_\theta^2 = \int_\Omega \left\{ |\Delta u|^2 + |B_\theta^{\frac{1}{2}} v|^2 + \frac{\alpha}{\beta} |w|^2 \right\} dx.$$

Gevrey regularity

Set $Z = \begin{pmatrix} y \\ y_t \\ z \end{pmatrix}$ and $Z^0 = \begin{pmatrix} y^0 \\ y^1 \\ z^0 \end{pmatrix}$.

The System may be rewritten as:

$$\begin{cases} \dot{Z} - \mathcal{A}_\theta Z = 0 & \text{in } (0, \infty), \\ Z(0) = Z^0, \end{cases} \tag{1}$$

where the unbounded operator matrix $\mathcal{A}_\theta$ is given by

$$\mathcal{A}_\theta = \begin{pmatrix} 0 & I & 0 \\ -B_\theta^{-1}\Delta^2 & 0 & -\alpha B_\theta^{-1}\Delta \\ 0 & \beta\Delta & \kappa\Delta \end{pmatrix}$$

Gevrey regularity

Theorem 1: Gevrey regularity

For every $\theta \in [0, 1]$, the linear operator $\mathcal{A}_\theta$ generates a strongly continuous-semigroup of contractions $(S_{\theta,j}(t))_{t\geq 0}$ on the Hilbert space $\mathcal{H}_{\theta,j}$, $j = 1,\ 2$.
Furthermore, for $j = 1,\ 2$, and for every θ in $(0, 1/2)$ the semigroup $(S_{\theta,j}(t))_{t\geq 0}$ is of Gevrey class δ for every $\delta > \frac{2-\theta}{2-4\theta}$, as there exists a positive constant C such that we have the resolvent estimate:

$$|\lambda|^{\frac{2-4\theta}{2-\theta}} ||(i\lambda I - \mathcal{A}_\theta)^{-1}||_{\mathcal{L}(\mathcal{H}_{\theta,j})} \leq C, \quad \forall\ \lambda \in \mathbb{R}. \tag{2}$$

Thus, the semigroup $(S_{\theta,j}(t))_{t\geq 0}$ is infinitely differentiable on $\mathcal{H}_{\theta,j}$ for all $t > 0$.

Gevrey regularity: Proof

We rely on sufficient condition involving the resolvent estimate along the imaginary axis provided by Taylor in his Thesis (UMN, 1989). Let $b \in \mathbb{R}$ with $|b| > 1$, $U = (f, g, h) \in \mathcal{H}_{\theta,j}$, and let $Z = (u, v, w) \in D(\mathcal{A}_\theta)$ such that

$$(ib - \mathcal{A}_\theta)Z = U.$$

It suffices to prove:

$$|b|^{\frac{2-4\theta}{2-\theta}} ||Z||_\theta \leq C_0 ||U||_\theta, \quad \forall b \in \mathbb{R} \text{ with } |b| > 1.$$

Main ideas

1) Rewrite $(ib - \mathcal{A}_\theta)Z = U$ as

$$\begin{cases} ibu - v = f, \\ ibB_\theta v + \Delta^2 u + \alpha \Delta w = B_\theta g, \\ ibw - \kappa \Delta w - \beta \Delta v = h. \end{cases}$$

2) Decompose the heat component as: $w = w_1 + w_2$, with

$$ibw_1 - \Delta w_1 = h, \quad ibw_2 = \kappa \Delta w + \beta v - \Delta w_1$$

Main ideas

One checks

$$|b||w_1|_2 + |b|^{\frac{1}{2}}||w_1||_W + |\Delta w_1|_2 \leq C_0||U||_\theta.$$

and

$$|b|||w_2||_{H^{-2}(\Omega)} \leq C_0(|w|_2 + |v|_2 + |w_1|_2) \leq C_0(||Z||_\theta + |b|^{-1}||U||_\theta).$$

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Lions' interpolation

$$|b||w_2|_2 \leq C_0|b|^{\frac{2}{3}}||bw_2||_{H^{-2}(\Omega)}^{\frac{1}{3}}||w_2||_{H^1(\Omega)}^{\frac{2}{3}}$$

leads to

$$|b|^{\frac{2-4\theta}{2-\theta}}|w|_2 \leq \varepsilon|b|^{\frac{2-4\theta}{2-\theta}}||Z||_\theta + C_\varepsilon||U||_\theta, \quad \forall\varepsilon > 0.$$

Main ideas

3) One decomposes the velocity component as $v = v_1 + v_2$ with

$$ibB_\theta v_1 - \Delta v_1 = B_\theta g, \quad ibB_\theta v_2 = -\Delta^2 u - \alpha \Delta w - \Delta v_1.$$

Proceeding as in the case of the heat component, one checks

$$|b||B_\theta^{\frac{1}{2}} v_1|_2 + |b|^{\frac{1}{2}} ||v_1||_W \leq C_0 ||U||_\theta$$

and

$$|b|||v_2||_{H^{2\theta-2}(\Omega)} \leq C_0(|\Delta u|_2 + |w|_2 + |v_1|_2) \leq C_0(||Z||_\theta + |b|^{-1}||U||_\theta).$$

Main ideas

Using interpolation inequalities, one then derives

$$|b|^{\frac{2-4\theta}{2-\theta}}|B_\theta^{\frac{1}{2}}v_2|_2 \leq \varepsilon|b|^{\frac{2-4\theta}{2-\theta}}||Z||_\theta + C_\varepsilon||U||_\theta, \quad \forall\varepsilon > 0,$$

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4) Finally, using the equation $iB_\theta v + \Delta^2 u + \alpha\Delta w = B_\theta g$, one shows

$$|b|^{\frac{2-4\theta}{2-\theta}}|\Delta u|_2 \leq \varepsilon|b|^{\frac{2-4\theta}{2-\theta}}||Z||_\theta + C_\varepsilon||U||_\theta, \quad \forall\varepsilon > 0.$$

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The claimed estimate then follows.

Lack of Analyticity

Theorem 2: Lack of Analyticity

Assume that the hinged boundary conditions hold. For every $\theta \in [0, 1]$, the strongly continuous semigroup $(S_{\theta,1}(t))_{t\geq 0}$ is exponentially stable on the Hilbert space $\mathcal{H}_{\theta,1}$. More precisely, there exist positive constants M and ω such that:

$$||S_{\theta,1}(t)||_{\mathcal{L}(\mathcal{H}_{\theta,1})} \leq M \exp(-\omega t).$$

However, the semigroup $(S_{\theta,1}(t))_{t\geq 0}$ is not analytic; more precisely, for every θ in $(0, 1]$, and every r in $((2-2\theta)/(2-\theta), 1]$, we have :

$$\limsup_{|\lambda|\to\infty} |\lambda|^r ||(i\lambda I - \mathcal{A}_\theta)^{-1}||_{\mathcal{L}(\mathcal{H}_{\theta,1})} = \infty.$$

Proof of lack of analyticity

Let $\theta \in (0, 1]$. Let $r \in ((2 - 2\theta)/(2 - \theta), 1]$. We are going to show that there exist a sequence of positive real numbers $(b_n)_{n\geq 1}$, and for each n, an element $Z_n \in \mathcal{D}(\mathcal{A}_\theta)$ such that:

$$\lim_{n\to\infty} b_n = \infty, \quad ||Z_n||_\theta = 1, \quad \lim_{n\to\infty} b_n^{-r}||(ib_n - \mathcal{A}_\theta)Z_n||_\theta = 0.$$

Indeed, if we have sequences b_n and Z_n as above, then we set

$$V_n = b_n^{-r}(ib_n - \mathcal{A}_\theta)Z_n, \qquad U_n = \frac{V_n}{||V_n||_\theta}.$$

Therefore $||U_n||_\theta = 1$ and

$$\lim_{n\to\infty} b_n^r||(ib_n - \mathcal{A}_\theta)^{-1}U_n||_\theta = \lim_{n\to\infty} \frac{1}{||V_n||_\theta} = \infty,$$

which would establish the claimed result, thereby completing the proof of Theorem 2.

Lack of analyticity

For each $n \geq 1$, we introduce the eigenfunction e_n, with $|e_n|_2 = 1$ and:

$$\begin{cases} -\Delta e_n = \omega_n e_n \text{ in } \Omega \\ e_n = 0 \text{ on } \partial\Omega. \end{cases}$$

It is a well-known fact that (ω_n) is an increasing sequence of positive real numbers with $\lim_{n\to\infty} \omega_n = \infty$. We seek Z_n in the form $Z_n = (a_n e_n, i b_n a_n e_n, c_n e_n)$, with b_n and the complex numbers a_n and c_n chosen such that Z_n fulfills the desired conditions.

Lack of Analyticity

Let γ and μ be two nonzero real numbers satisfying:

$$2(\gamma^2 + \mu^2) = 1.$$

For some large enough N_θ, to be specified later, and for $n \geq N_\theta$, set:

$$b_n = \frac{\omega_n}{\sqrt{1 + \omega_n^\theta}}, \quad \omega_n a_n = \mu_n + i\gamma_n, \quad c_n = -\frac{i\beta\omega_n a_n}{i + \kappa\sqrt{1 + \omega_n^\theta}}.$$

Now, we shall specify what μ_n and γ_n are. To this end, introduce the sequence of real numbers

$$r_n = -\frac{\alpha\beta}{4(2(1 + \kappa^2(1 + \omega_n^\theta)) + \alpha\beta)}.$$

Lack of Analyticity

Now, r_n goes to zero as n goes to infinity, for any positive θ, and γ and μ are nonzero, there exists a positive integer M_θ such that both numbers $\gamma^2 + r_n$ and $\mu^2 + r_n$ are positive for each $n \geq M_\theta$; we choose $N_\theta = M_\theta$, and set

$$\mu_n = \pm\sqrt{\mu^2 + r_n}, \quad \gamma_n = \pm\sqrt{\gamma^2 + r_n}.$$

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$$\mu_n = \pm\sqrt{\mu^2 + r_n}, \quad \gamma_n = \pm\sqrt{\gamma^2 + r_n}.$$

With that choice, we have for every $n \geq N_\theta$:

$$\begin{aligned}
||Z_n||_\theta^2 &= \omega_n^2|a_n|^2 + b_n^2(1 + \omega_n^\theta)|a_n|^2 + \frac{\alpha}{\beta}|c_n|^2 \\
&= 2(\mu_n^2 + \nu_n^2) + \frac{\alpha\beta(\mu_n^2 + \gamma_n^2)}{1 + \kappa^2(1 + \omega_n^\theta)} \\
&= 2(\mu^2 + \gamma^2) + 4r_n + \frac{\alpha\beta(\mu^2 + \gamma^2 + 2r_n)}{1 + \kappa^2(1 + \omega_n^\theta)} \\
&= 1.
\end{aligned}$$

Lack of Analyticity

$$(ib_n - \mathcal{A}_\theta)Z_n = \begin{pmatrix} 0 \\ \left(\left[(\omega_n^2 - b_n^2(1+\omega_n^\theta))a_n - \alpha\omega_n c_n\right] B_\theta^{-1} e_n \right. \\ i\beta b_n\omega_n a_n e_n + (ib_n + \kappa\omega_n)c_n e_n \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -\alpha\omega_n c_n B_\theta^{-1} e_n \\ 0 \end{pmatrix},$$

Lack of Analyticity

$$
\begin{aligned}
&\lim_{n\to\infty} b_n^{-2r} ||(ib_n - \mathcal{A}_\mu)Z_n||_\theta^2 \\
&= \lim_{n\to\infty} \alpha^2 b_n^{-2r} \omega_n^2 |c_n|^2 |B_\theta^{-\frac{1}{2}} e_n|_2^2 = \lim_{n\to\infty} \frac{\alpha^2 b_n^{-2r} \omega_n^2 |c_n|^2}{1+\omega_n^\theta} \\
&= \lim_{n\to\infty} \frac{\alpha^2 \beta^2 b_n^{-2r} \omega_n^2 (\mu_n^2 + \gamma_n^2)}{(1+\kappa^2(1+\omega_n^\theta))(1+\omega_n^\theta)} = \lim_{n\to\infty} \frac{\alpha^2 \beta^2 \omega_n^{-2r+\theta r} \omega_n^{2-2\theta}}{2\kappa^2} \\
&= 0,
\end{aligned}
$$

provided that

$$
2 - 2\theta - r(2-\theta) < 0, \text{ or } \; r > \frac{2-2\theta}{2-\theta}.
$$

Model formulation

$$\begin{cases} B_\theta y_{tt} + a\Delta^2 y + b(-\Delta)^\delta y_t = 0 \text{ in } \Omega \times (0,\infty), \\ y(0) = y^0 \in V, \quad y_t(0) = y^1 \in H_\theta \end{cases}$$

where a and b are positive constants, θ and δ are nonnegative constants with

$$\theta \in [0,1] \text{ and } \delta \in [0,2].$$

The operator B_θ is given by $B_\theta u = u + (-\Delta)^\theta u$, and $H_\theta = D(B_\theta^{\frac{1}{2}})$. The boundary conditions are:

(Hinged plate): $y = 0, \quad \Delta y = 0$ on $\Sigma = \partial\Omega \times (0,\infty)$, or, else

(Clamped plate): $y = 0, \quad \partial_\nu y = 0$ on $\Sigma = \partial\Omega \times (0,\infty)$.

Brief literature

Responding to two conjectures of Chen and Russell (1982), Chen and Triggiani considered the following abstract evolution system

$$y_{tt} + Ay + By_t = 0 \text{ in } (0, \infty)$$

where A and B are unbounded operators of some Hilbert space H with

$$\alpha_1 A^{\mu} \leq B \leq \alpha_2 A^{\mu}, \text{ for some constant } \mu \in (0, 1],$$

.

They proved that the underlying semigroup is

1. analytic for $\frac{1}{2} \leq \mu \leq 1$, but not analytic for $0 < \mu < \frac{1}{2}$, though differentiable, (1989)
2. of Gevrey class δ for all $\delta > \frac{1}{2\mu}$ for $0 < \mu < \frac{1}{2}$, (1990)

In fact, the Gevrey class result generalizes the work of Taylor (1989), where the author discusses Gevrey semigroups, and illustrates his work with several examples including the case $B = 2\rho A^{\mu}$ for some positive constant ρ.

Analyticity

Theorem 3

For every $\theta \in [0,1]$ and $\delta \in [0,2]$ the linear operator $\mathcal{A}_{\theta,\delta,j}$, ($j = 1,\ 2$), associated with the plate equation, generates a strongly continuous semigroup of contractions $(S_{\theta,\delta,j}(t))_{t\geq 0}$ on the Hilbert space $\mathcal{H}_{\theta,j}$, $j = 1,\ 2$.
Furthermore, for $j = 1,\ 2$, we have: For every θ in $[0,1]$ and every δ in $[1+(\theta/2), 2]$, the semigroup $(S_{\theta,\delta,j}(t))_{t\geq 0}$ is analytic, as there exists a positive constant C such that the following resolvent estimate holds:

$$|\lambda| \left\| (i\lambda I - \mathcal{A}_{\theta,\delta,j})^{-1} \right\|_{\mathcal{L}(\mathcal{H}_{\theta,j})} \leq C, \quad \forall\ \lambda \in \mathbb{R}.$$

Gevrey regularity

Theorem 4

For every θ in $[0,1]$ and every δ in $(\theta, 1+(\theta/2))$, the semigroup $(S_{\theta,\delta,j}(t))_{t\geq 0}$ is of Gevrey class α for every $\alpha > \frac{2-\theta}{2(\delta-\theta)}$, as there exists a positive constant C such that we have the resolvent estimate:

$$|\lambda|^{\frac{2(\delta-\theta)}{2-\theta}} \left\| (i\lambda I - \mathcal{A}_{\theta,\delta,j})^{-1} \right\|_{\mathcal{L}(\mathcal{H}_{\theta,j})} \leq C, \quad \forall\, \lambda \in \mathbb{R}.$$

Lack of analyticity

Theorem 5

Assume that the hinged boundary conditions hold. Then for every $\theta \in (0, 1]$, and every δ in $(\theta, 1 + (\theta/2))$, the semigroup $(S_{\theta,\delta,1}(t))_{t\geq 0}$ is not analytic on the Hilbert space $\mathcal{H}_{\theta,1}$. More precisely, for every r in $(2(\delta - \theta)/(2 - \theta), 1]$, we have :

$$\limsup_{|\lambda|\to\infty} |\lambda|^r \left\| (i\lambda I - \mathcal{A}_{\theta,\delta,1})^{-1} \right\|_{\mathcal{L}(\mathcal{H}_{\theta,1})} = \infty.$$

Exponential stability

Theorem 6

For every $\theta \in [0, 1]$, and every δ in $[\theta, 2]$, the strongly continuous semigroup $(S_{\theta,\delta,j}(t))_{t\geq 0}$ is exponentially stable. More precisely, there exist positive constants K and ξ such that:

$$||S_{\theta,\delta,j}(t)||_{\mathcal{L}(\mathcal{H}_{\theta,j})} \leq K \exp(-\xi t), \quad \forall\, t \geq 0.$$

Proof method: frequency method based on Huang (1985) or Pruss (1984) exponential stability criterion.

Polynomial stability

Theorem 7

For every $\theta \in (0,1]$, and every δ in $[0,\theta)$, the semigroup $(S_{\theta,\delta,j}(t))_{t\geq 0}$ is not exponentially stable. However, the semigroup is polynomially stable, so that there exists a positive constant K_0 with:

$$||S_{\theta,\delta,j}(t)Z^0||_{\mathcal{L}(\mathcal{H}_{\theta,1})} \leq \frac{K_0||Z^0||_{D(\mathcal{A}_{\theta,\delta,j})}}{(1+t)^{\frac{2-\theta}{2(\theta-\delta)}}}, \quad \forall Z^0 \in D(\mathcal{A}_{\theta,\delta,j}), \quad \forall\, t \geq 0.$$

Furthermore, in the hinged boundary conditions case, the polynomial decay rate is optimal. More precisely, for every r in $[0, 2(\theta-\delta)/(2-\theta))$, we have :

$$\limsup_{|\lambda|\to\infty} |\lambda|^{-r} \left\|(i\lambda I - \mathcal{A}_{\theta,\delta,1})^{-1}\right\|_{\mathcal{L}(\mathcal{H}_{\theta,1})} = \infty.$$

Proof method: frequency method based on Borichev-Tomilov (2010)

Some comments

In the first part of this talk, we are able to prove that the semigroup corresponding to the thermoelastic plate model with rotational forces involving the spectral fractional Laplacian is of Gevrey class $\delta > (2-\theta)/(2-4\theta)$ for all θ in $(0, 1/2)$; this is also valid for the hinged or clamped plate. The case θ in $[1/2, 1)$ remains open, though Theorem 2 shows that the semigroup fails to be analytic for all θ in $(0, 1]$ in the case of a hinged plate. Our work shows that analyticity occurs only for the Euler-Bernoulli model, corresponding to $\theta = 0$, at least for hinged boundary conditions.

Some comments

Though Theorem 2 shows exponential stability of the semigroup for hinged boundary conditions for the plate and all values of $\theta \in [0, 1]$, in the case of clamped boundary conditions, we were able to get the exponential stability of the semigroup only for $\theta \in [0, 1/2]$, by using our resolvent estimate; the case $\theta \in (1/2, 1]$ remains open. Also, the regularity issue for the most general case where all operators are fractional remains open; see Liu-Racke (2020) for the stability issue in the Euler-Bernoulli case, with the fractional operators appearing in the coupling.

Some comments

In the second part of the talk, we state the following results: the underlying semigroup is:

1. analytic for all θ in $[0,1]$ and δ in $[(2+\theta)/2, 2]$,
2. not analytic for all θ in $[0,1]$ and δ in $(\theta, (2+\theta)/2)$,
3. of Gevrey class $\alpha > (2-\theta)/2(\delta-\theta)$ for all θ in $[0,1]$ and $\theta < \delta < (2+\theta)/2$.

We also prove that for each admissible value of θ, the semigroup is exponentially stable for $\delta \geq \theta$, and only polynomially stable, with rate $O(t^{-\frac{2-\theta}{2(\theta-\delta)}})$, for $\delta < \theta$, when $\theta > 0$. In particular, in the hinged plate case, we show that the polynomial decay rate is optimal, and we also show that the resolvent estimates for both analyticity and Gevrey class regularity are sharp.

And if anyone thinks that he knows anything, he knows nothing yet as he ought to know.

THANKS!