MAC 2311 (Calculus I)
TEST 1, Friday October 02, 2009

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good Luck.

1. [40] Evaluate the following limits (Show all your work)

a) $\lim_{x\to 1} \frac{x^2-3x}{x^3-2x+6} =$

b) $\lim_{x\to +\infty} \cos\left(\frac{-\pi x^4+3x+7}{8-5x^2+2x^4}\right) =$

c) $\lim_{x\to -2^-} \frac{x}{x+2} =$

d) $\lim_{x\to 2} \frac{\sqrt{3x-2}-2}{x-2} =$

e) $\lim_{x\to -3} \sqrt{\frac{3x^2-5x+3}{-5x-3}} =$

f) $\lim_{x\to \frac{\pi}{4}^-} \frac{\sin(2x)}{|x|} =$

g) $\lim_{x\to 3} \frac{x^2-9}{x^2-5x+6} =$

h) $\lim_{x\to -\infty} \frac{\sqrt{9x^2-5x+6}}{-5x+7} =$

2. [7] If $f(x) = \begin{cases} 2x^2 - 3, & x > 1 \\ x^3 - 2, & x \le 1. \end{cases}$

Is f continuous at $x = 1$? You must carefully explain your answer to get any credits.

3. [10] Write down the rigorous definition of $\lim_{x \to a} f(x) = L$, and use it to prove that $\lim_{x \to -3} (5x + 6) = -9$.

4. [6] a) Use the implicit differentiation technique to find $\frac{dy}{dx}$ if $x^2y^3 - 4x + 12y = 8$.

b) Find the equation of the tangent line to the curve $x^2y^3 - 4x + 12y = 8$ at the point (2,1).

5. [37] Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit by guessing the correct answer(s).)

a) $f(x) = 2x^3 - \frac{5}{\sqrt[3]{x}} + \frac{7}{x^2}$

b) $g(x) = \frac{3x-4}{x^2+x+1}$

c) $h(x) = x^2 \ln(x)$

d) $k(x) = \sin^2(\sec^3 x) + \cos^2(\sec^3 x)$

e) $l(x) = \tan(x^3 - \csc x)$

f) $m(x) = \sin(\cos x)$

g) Use the logarithmic differentiation technique to find the derivative of $p(x) = \frac{x^2 \sqrt[5]{x^2-x+2}}{x^3+x-1}$.

Bonus. [6] $\lim_{x \to -\infty} (\sqrt{4x^2 - 5x} + 2x) =$