MAC 2311 (Calculus I)
Test 1, Monday October 17, 2011

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University administration. Good Luck!

1. [30] Evaluate the following limits (Show all your work. You cannot use de l'Hopital's rule for any of the limits a) to f), otherwise you'll get a zero. You will not get any credit(s) by guessing the correct answer(s).)

a) $\displaystyle\lim_{x\to-1}\frac{x^4-3x+2}{2x^3+5x-8}=$

b) $\displaystyle\lim_{x\to-\infty}\frac{5x^5-7x^3+10x+10^{12}}{2x-x^6+5}=$

c) $\displaystyle\lim_{x\to2\pi}\frac{\sin x}{x}=$

d) $\displaystyle\lim_{x\to1}\frac{\sqrt{x}-1}{x^2-1}=$

e) $\displaystyle\lim_{x\to-5^-}\frac{x+3}{x+5}=$

f) $\displaystyle\lim_{x\to0}\frac{\sin^2(4x)}{x^2}=$

2. [10] Use de l'Hopital's rule to find the following limits.

i) $\displaystyle\lim_{x\to0}\frac{\sin x}{e^x-1}=$

ii) $\displaystyle\lim_{x\to+\infty}\frac{e^x}{x}=$

3. [10] a) Write down the two definitions for $f'(x_0)$. b) Use any of those definitions to find $f'(2)$ if $f(x)=\frac{1}{x}$. c) Use b) to find the equation of the tangent line to the curve $y=1/x$ at $x=2$.

4. [10] a) Write down the rigorous definition of $\displaystyle\lim_{x\to-3}f(x)$. b) Use that definition to show that $\displaystyle\lim_{x\to-3}(-4x+1)=13$.

5. [32] Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit(s) by guessing the correct answer(s).)

a) $f(x) = \frac{x^2-3x+1}{x^2+x-2}$

b) $g(x) = x\cos(x^2)$

c) $h(x) = \cos^{-1}(3x)$

d) $k(x) = e^{\tan x}$

e) Use the logarithmic differentiation technique to find $\frac{dy}{dx}$ if $y = (\sin x)^{x^3}$.

f) Use the implicit differentiation technique to find $\frac{dy}{dx}$ if $x^3 - \cos(y) = y$

6. [14] Decide whether the statement is true or false. No explanation needed.

a) If $\lim_{x\to -2} \dfrac{f(x) - f(-2)}{x+2} = 17$, then $\lim_{h\to 0} \dfrac{f(-2+h) - f(-2)}{h} = 17$.

b) If f is differentiable at a, then f is continuous at a.

c) If f is continuous at x_0, then $\lim_{x\to x_0} f(x) = f(x_0)$.

d) If $f(-3) = 5$, then $\lim_{x\to -3} f(x) = 5$.

e) If $\lim_{x\to x_0^+} f(x) = 26$ and $\lim_{x\to x_0^-} f(x) = 26$, then f is continuous at x_0.

f) If $|f|$ is continuous at -1, then f is continuous at -1.

g) $\lim_{x\to +\infty} (x - x^2) = +\infty - (+\infty) = 0$.