MAC 2312 (Calculus II)
Test 1, Thursday September 28, 2006

Name: **PID:**

Remember that no documents or graphing calculators are allowed during the test. Be as precise as possible in your work; you shall show all your work to deserve any of the points assigned to any question. *You will not get any credit by just writing down the answer to any of the problems.* **Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.**

1. [50] Computation of integrals; exact value shall be provided for each integral.

a) $\int_{-1}^{1} x(1-x^2)\,dx = \int_{-1}^{1}(x-x^3)\,dx = \frac{x^2}{2}-\frac{x^4}{4}\Big]_{-1}^{1} = \frac{1}{2}-\frac{1}{4}-\left(\frac{1}{2}-\frac{1}{4}\right)=0$

b) $\int_1^4 |2x-4|\,dx = 2\int_1^2 |x-2|\,dx + 2\int_2^4 |x-2|\,dx$

$= 2\int_1^2 (2-x)\,dx + 2\int_2^4 (x-2)\,dx$

$= -(2-x)^2\Big]_1^2 + (x-2)^2\Big]_2^4$

$= 0 + 1 + 2^2 - 0$

$= 5$

c) $\int_0^{\frac{\pi}{2}} \cos x\,dx = \sin x\Big]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0 = 1-0 = 1$

d) $\int_0^2 (x+e^x)\,dx = \frac{x^2}{2} + e^x\Big]_0^2 = \frac{4}{2} + e^2 - 1 = 1 + e^2$

e) $\int_0^5 \frac{dx}{\sqrt{3x+1}} = \int_1^4 (2/3)\,du = \frac{2}{3} u \Big]_1^4 = \frac{2}{3}(4-1) = 2.$

Set $u = \sqrt{3x+1}$; $du = \frac{3}{2\sqrt{3x+1}} dx$

<u>Method 2</u>: $\int \frac{dx}{\sqrt{3x+1}} = \frac{2}{3} \int du = \frac{2}{3} u$ (No constant needed as we are dealing with definite integral)

$= \frac{2}{3}\sqrt{3x+1}$

$\int_0^5 \frac{dx}{\sqrt{3x+1}} = \frac{2}{3}\sqrt{3x+1}\Big|_0^5 = \frac{2}{3}(\sqrt{16} - \sqrt{1}) = \frac{2}{3}(4-1) = 2.$

f) $\frac{d}{dx}\int_0^x \tan(3+\ln(4+t^2))\,dt =$

$\to \tan(3+\ln(4+x^2))$ by FTC2

g) $\int_0^{\frac{\pi}{3}} \sec x \tan x\,dx =$ $\sec x \Big]_0^{\pi/3} = \sec\frac{\pi}{3} - \sec 0 = 2 - 1 = 1$

h) $\int_1^6 \frac{dr}{r} =$ $\ln r \Big]_1^6 = \ln 6 - \ln 1 = \ln 6$

i) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2 x\,dx =$ $-\cot \Big]_{\pi/4}^{\pi/2} = -\cot\frac{\pi}{2} + \cot\frac{\pi}{4} = -0 + 1 = 1$

j) $\int_0^1 x\sqrt{(5x^2+4)}\,dx =$ $\int_4^9 \sqrt{u}\cdot\frac{du}{10} = \frac{1}{10}\int_4^9 u^{1/2}du = \frac{1}{10}\cdot\frac{2}{3}\, u^{3/2}\Big]_4^9 = \frac{1}{15}\left[27-8\right] = \frac{19}{15}$

Set $u = 5x^2+4$; $du = 10x\,dx$

2. [35] Evaluate the indefinite integrals.

i) $\int (x^2-3x+7)e^{-x}\,dx =$ $-(x^2-3x+7)e^{-x}$
$-(2x-3)e^{-x}$
$-2e^{-x}+C$

Tabular integration by parts

Derivatives of p		Integrals of f
x^2-3x+7	$+$	e^{-x}
$2x-3$	$-$	$-e^{-x}$
2	$+$	e^{-x}
0		$-e^{-x}$

ii) $\int \sin^2 x\cos^3 x\,dx =$ $\int \sin^2 x\cos^2 x\cos x\,dx$, set $u=\sin x$; $du = \cos x\,dx$

$= \int u^2(1-u^2)\,du$

$= \int (u^2-u^4)\,du$

$= \frac{u^3}{3} - \frac{u^5}{5} + C = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

iii) $\int \tan^3 x\sec^2 x\,dx =$ $\int u^3 du = \frac{u^4}{4} + C = \frac{\tan^4 x}{4} + C$

Set $u = \tan x$; $du = \sec^2 x\,dx$

iv) $\int \frac{x^2+1}{x^2+4}\,dx =$ $\int \frac{x^2+4-3}{x^2+4}\,dx$

$= \int 1 - \frac{3}{x^2+4}\,dx$

$= x - 3\int\frac{dx}{x^2+4}$,

$= x - \frac{3}{2}\tan^{-1}(x/2) + C$

set $x = 2\tan u$; $dx = 2(1+\tan^2 u)\,du$

$\int\frac{dx}{x^2+4} = \int\frac{2(1+\tan^2 u)\,du}{4(\tan^2 u+1)} = \frac{1}{2}\int du$

$= \frac{u}{2} + C$

$= \frac{\tan^{-1}(x/2)}{2} + C$

v) $\int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{4+4x-x^2-4}}$

$= \int \frac{dx}{\sqrt{4-(x-2)^2}}$, Set $x-2 = 2\sin\theta$; $dx = 2\cos\theta\, d\theta$

$= \int \frac{2\cos\theta\, d\theta}{\sqrt{4(1-\sin^2\theta)}} = \int \frac{\cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}} = \int d\theta = \theta + C = \sin^{-1}\left(\frac{x-2}{2}\right) + C$

vi) $\int \frac{x+1}{x^2-3x+2}\, dx = \int \frac{(x+1)\, dx}{(x-1)(x-2)} = \int \left(\frac{A}{x-1} + \frac{B}{x-2}\right) dx = -2\ln|x-1| + 3\ln|x-2| + C$

$= \ln \frac{|x-2|^3}{(x-1)^2} + C$

$\frac{x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$

So $A(x-2) + B(x-1) = x+1$

Set $x=1$ to get $-A = 2 \rightarrow A = -2$

Set $x=2$ to get $B = 3$

vii) $\int \frac{dx}{(x+2)(x^2+9)} = \frac{1}{13}\int \frac{dx}{x+2} + \int \frac{-\frac{1}{13}x + \frac{2}{13}}{x^2+9} dx = \frac{1}{13}\ln|x+2| - \frac{1}{26}\ln(x^2+9) + \frac{2}{13}\int \frac{dx}{x^2+9}$

$\frac{1}{(x+2)(x^2+9)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+9} = \frac{A(x^2+9)+(Bx+C)(x+2)}{(x+2)(x^2+9)}$

Set $x = 3\tan\theta$; $dx = 3(1+\tan^2\theta)d\theta$

$\int \frac{dx}{x^2+9} = \int \frac{3(1+\tan^2\theta)d\theta}{9(1+\tan^2\theta)}$

$= \frac{\theta}{3} + C = \frac{\tan^{-1}(x/3)}{3} + C$

So $A(x^2+9) + (Bx+C)(x+2) = 1$

Set $x = -2$ to get $A = \frac{1}{13}$

Set $x = 3i$ to get $(3iB+C)(3i+2) = 1$

$-9B + 2C + i(6B+3C) = 1 \Rightarrow 6B + 3C = 0$, $-9B+2C = 1$; hence $B = -1/13$ $C = 2/13$

3. [15] a) State the first part of the Fundamental Theorem of Calculus (FTC1). b) Thoroughly explain whether or not the FTC1 may be used to evaluate each of the integrals (do not attempt to evaluate these integrals): b1) $\int_0^\pi \frac{dx}{1+(\cos x)^{92806}}$, b2) $\int_1^3 (1+(x-2)^{27})\ln(3-x)\, dx$.

a) see class notes or text.

b1) the function $f: x \mapsto f(x) = \frac{1}{1+(\cos x)^{9282006}}$ is continuous on $[0,\pi]$

so f satisfies the requirement of the FTC1; so we can use the FTC1 to evaluate $\int_0^\pi f(x)dx$.

b2) If we set $g(x) = (1+(x-2)^{27})\ln(3-x)$, then

$\lim_{x\to 3^-} g(x) = 2\lim_{x\to 3^-}\ln(3-x) = -\infty$; so g is not continuous on $[1,3]$; we cannot use the FTC1 to evaluate $\int_1^3 g(x)dx$.