**MAC 2312 (Calculus II)**
**Test 1, Wednesday February 22, 2012**

**Name:** **PID:**

**Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; no credits will be awarded for unexplained answers. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.**

1. [6] Determine whether the improper integral converges or diverges. If it converges, state its limit, and if it diverges, state whether it diverges to $+\infty$, to $-\infty$, or due to oscillations.

$$\int_1^{+\infty} \frac{dx}{x^{-\frac{22}{3}}}\, dx =$$

2. [6] A particle moving along a straight line is accelerating at a constant rate of $4m/s^2$. Find the initial velocity if the particle moves $36m$ in the first $3s$.

3. [8] a) Use the difference $a_{n+1} - a_n$ to show that the sequence $(a_n)_n$ given by: $a_n = \dfrac{3n}{4n+5}, \quad n = 1,\ 2,\ ...,$ is strictly increasing. b) Derive from a) that the sequence $(a_n)_n$ converges. c) Find its limit.

4. [10] Decide whether each statement is true or false. No explanation is needed.

a) If $f$ is integrable on $[a,b]$, then $f$ is continuous on $[a,b]$.

b) If $(a_n)$ is a decreasing sequence, then $(a_n)$ converges.

c) If $f$ is integrable on $[a,b]$, then $\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = 0$

d) $\displaystyle\int_{-2}^{2} \frac{dx}{x^3} = 0.$

e) If $(a_n)$ is a bounded sequence, then $(a_n)$ converges.

5. [8] Approximate the integral $\int_0^2 \cos(x^3)\,dx$ using: a) the midpoint rule with $n = 2$. b) Simpson's rule with $n = 2$.

6. [15] Evaluate each definite integral.

a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (t - \frac{3}{\sin^2 t})\,dt =$

b) $\int_1^4 \frac{|3-x|}{x}\,dx =$

c) $\int_{1-\pi}^{1+\pi} \sec^2(\frac{1}{3} - \frac{u}{3})\,du =$

7. [10] a) Write the expression in sigma notation, but do not evaluate.

$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} =$

b) Use the values $a = 1$ and $b = 2$ to express the limit as an integral. Do not evaluate the integral.

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} \ln(1 + e^{x_k^*})\Delta x_k =$$

8. [25] Evaluate each indefinite integral.

i) $\int \sin x \cos(2x)\, dx =$

j) $\int e^{\tan v} \sec^2 v\, dv =$

k) $\int \frac{\ln t}{t^3}\, dt =$

l) $\int \frac{2x^2-1}{x(x^2+1)}\, dx =$

9. [10] a) Find the derivative $F'(x)$ if $F(x) = \int_{x^2}^{\cos x} \sin(\pi t + t^3)\, dt$.

b) Use the definition of the definite integral to write the given integral as the limit of a Riemann sum. Do not evaluate the integral.

$\int_0^{\frac{\pi}{4}} \sin(\cos x)\, dx =$

10. [5] State the fundamental theorem of calculus, part 1.