MAP 2302 (Differential Equations)
TEST 1, Thursday February 11, 2010

Name: **PID:**

Remember that no documents or calculators are allowed during the test. You shall show all your work to deserve the full mark assigned to any question. 5 pages. Total=100 points

1. [11+10] a) Show that the function given by $f(x) = 2e^{-2x} + \sin x - \cos x$ is the solution of the initial-value problem: $y'' + 4y' + 5y = 2e^{-2x} + 8\sin x, \quad y(0) = 1, \quad y'(0) = -3$. b) Show that the differential equation: $(x^2y + 2y - 3)dx + xdy = 0$ is not exact. b1) Find an integrating factor for that equation. b2) Write down the exact differential equation, but do not solve it.

2. [10] State Theorem 1.1 from the text. Use that theorem to show that the initial-value problem:

$$\begin{cases} \dfrac{dy}{dx} = 2^{xy} - y^3x^2 \\ y(4) = \pi. \end{cases}$$

has a unique solution defined on some sufficiently small interval $|x-4| \leq h$ about $x_0 = 4$.

3. [12] Solve the initial-value problem: $(x^2+1)\dfrac{dy}{dx} + 2xy = x^3, \quad y(0) = 2.$

4. [15] Given that $y = x^2$ solves the differential equation: $x^2y'' - 6xy' + 10y = 0$, use the method of reduction of order to find a linearly independent solution. Write down the general solution.

5. [12+10] a) Solve the homogeneous differential equation: $(x^2+3y^2)dx - 2xydy = 0$.

b) Reduce the equation $(x+3y-7)dx + (4x+12y+8)dy = 0$ to a separable equation. Do not solve the separable equation obtained.

6. [10] Find the orthogonal trajectories to the family of curves $x^2 - 4y^2 = c$.

7. [10] Find the constant A such that the differential equation: $(3x^2y^2 + Ay)dx + (2x^3y + 4x - \sin y)dy = 0$ is exact. Solve the exact differential equation.