MAS 3105 (Linear Algebra)
Test 1, Friday May 27, 2016

Name: **PID:**

Remember that you won't get any credit if you do not show the steps to your answers. You may show your work on the back of each page. Total=105 points.

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1. [20] Solve the linear system

$$\begin{aligned} x_2 + x_3 + x_4 &= 0 \\ 3x_1 + 3x_2 - 4x_3 &= 9 \\ x_1 + x_2 + 2x_3 + x_4 &= 6 \\ 2x_1 + 3x_2 + x_3 + 3x_4 &= 6 \end{aligned}$$

2. [20] State whether each of the following statement is true or false.
No explanations needed.

(1) If A is an $n \times n$ matrix with $A^2 = 0_{\mathcal{M}_n}$, then A is singular.
(2) If A a 20×20 matrix that is row equivalent to a nonsingular matrix B, then $\det(A) \neq 0$.
(3) If A is an 11×15 matrix, then $A^T A$ is a 15×15 matrix.
(4) If U is a nonempty subset of a vector space E, then U is a subspace of E.
(5) If A, and B satisfy $\det(A) = \det(B)$, then $\det(AB) \geq 0$.
(6) If $A^2 - 3A + I_n = 0_{\mathcal{M}_n}$, then A is nonsingular.
(7) If an $m \times m$ matrix A satisfies $A^T = A$, then A is nonsingular.
(8) If A and B are $n \times n$ matrices, then $\det(AB) = \det(BA)$.
(9) If A and B are nonsingular $n \times n$ matrices, then $A + B$ is also nonsingular.
(10) If A and B are 15×15 matrices with $A = B^T$, then $\det(A) = \det(B)$.

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3. [15] Consider the linear system whose augmented matrix is $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & m & 3 & 0 \\ -1 & 1 & 5 & 0 \end{array}\right]$

a) Is it possible for this system to be inconsistent? Explain, or no credit.

b) For which value(s) on m will the system have infinitely many solutions? Write down the solution set(s) in this case.

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4. [10] For which values of the number a do we have $A_a^2 = I_2$ if $A_a = \begin{bmatrix} a-1 & 1 \\ -2 & 1-a \end{bmatrix}$, and I_2 denotes the identity matrix of order 2?

5. [20] Find the inverse of the matrix (Hint. You may use the reduction method.)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 1 & 3 & 5 \end{pmatrix}.$$

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6. [10] a) Let A be an $n \times n$ matrix. Set $B = A + A^T$ and $D = A^T - A$. Show that B is symmetric and D is skew symmetric.

b) Let A denote the matrix in problem 5. Find an upper triangular matrix U and a lower triangular matrix L such that $A = LU$.

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7. [10] a) Let V be a vector space, and let S be a subset of V. Complete the sentence: S is called a subspace of V when

b) Let A and B be $m \times m$ matrices with $AB = A + B$. Show that, if B is nonsingular, then A is nonsingular.