MAC 2313 (Calculus III)
Test 1 Review- Spring 2015

1. Describe the given surface; if it is a sphere, state its radius and center. If it is a point, state its coordinates. a) $x^2+y^2+z^2+6x-2y-6=0$. b) $x^2+y^2+z^2-2mx-6y-8z+50=0$, where m is a parameter. (discuss according to the values of m.)

2. a) Find an equation for the sphere passing through the origin and centered at the point $C(1,-2,5)$. b) Decide whether the points $A(2,3,1)$, $B(-1,1,-2)$ and $C(1,-1,1)$ are the vertices of an equilateral triangle.

3. Let $\overrightarrow{r}$=2$\overrightarrow{i}$-3$\overrightarrow{j}$+4$\overrightarrow{k}$, $\overrightarrow{z}$=3$\overrightarrow{j}$-5$\overrightarrow{k}$, and $\overrightarrow{v}$=-2$\overrightarrow{i}$+$\overrightarrow{j}$-4$\overrightarrow{k}$. a) Find the area of the parallelogram having $\overrightarrow{r}$ and $\overrightarrow{z}$ as adjacent sides. b) Find the volume of the parallelepiped having $\overrightarrow{r}$, $\overrightarrow{z}$ and $\overrightarrow{v}$ as adjacent edges. c) Find the acute angle θ between $\overrightarrow{v}$ and the plane containing the face determined by $\overrightarrow{r}$ and $\overrightarrow{z}$.

4. a) Find the volume of the parallelepiped having $\overrightarrow{u}$=2$\overrightarrow{i}$-3$\overrightarrow{j}$+7$\overrightarrow{k}$, $\overrightarrow{v}$=$\overrightarrow{i}$-2$\overrightarrow{j}$+$\overrightarrow{k}$, and $\overrightarrow{w}$=$\overrightarrow{i}$-3$\overrightarrow{j}$-$\overrightarrow{k}$ as adjacent edges. b) Find the area of the face determined by $\overrightarrow{u}$ and $\overrightarrow{v}$. c) Find the acute angle θ between $\overrightarrow{w}$ and the plane determined by $\overrightarrow{u}$ and $\overrightarrow{v}$.

5. a) Let $A(x_0,y_0,z_0)$ be a given point in 3-space. Let $\mathcal{P}$ be the plane with equation $ax+by+cz+d=0$. Write down the distance D between A and the plane $\mathcal{P}$. $D=$
b) Use a) to find the distance between the two parallel planes: $\mathcal{P}_1: 2x+3y-z=4$ and $\mathcal{P}_2: 2x+3y-z=-3$.

6. Let $\overrightarrow{w}$=$\overrightarrow{i}$-2$\overrightarrow{j}$+3$\overrightarrow{k}$ and $\overrightarrow{v}$=2$\overrightarrow{i}$-$\overrightarrow{j}$-$\overrightarrow{k}$. Find the vector component of $\overrightarrow{v}$ that is parallel to $\overrightarrow{w}$ and the vector component of $\overrightarrow{v}$ that is orthogonal to $\overrightarrow{w}$.

7. a) Set $\overrightarrow{u}$=$\overrightarrow{i}$-3$\overrightarrow{k}$, $\overrightarrow{v}$=-$\overrightarrow{j}$+$\overrightarrow{k}$ and $\overrightarrow{w}$=2$\overrightarrow{i}$-$\overrightarrow{j}$. Let $\overrightarrow{z}$=$\overrightarrow{i}$-$\overrightarrow{j}$+2$\overrightarrow{k}$. Find scalars a, b, and c such that $\overrightarrow{z}=a\overrightarrow{u}+b\overrightarrow{v}+c\overrightarrow{w}$. b) If we now set: $\overrightarrow{u}$=$\overrightarrow{i}$+$\overrightarrow{j}$-2$\overrightarrow{k}$, $\overrightarrow{v}$=$\overrightarrow{i}$+$\overrightarrow{j}$+$\overrightarrow{k}$ and $\overrightarrow{w}$=$\overrightarrow{i}$-$\overrightarrow{j}$, find scalars α, β and γ such that $\overrightarrow{z}=\alpha\overrightarrow{u}+\beta\overrightarrow{v}+\gamma\overrightarrow{w}$.

8. a) Find parametric equations for the line through the points $A(-1,2,3)$ and $B(2,-3,4)$. b) Find the vector $\overrightarrow{w}$ of norm 4 that is oppositely directed to $\overrightarrow{z}$=2$\overrightarrow{i}$-$\overrightarrow{j}$+3$\overrightarrow{k}$. c) Find parametric equations for the line through the point $A(5,0,-2)$ that is parallel to the planes $x-4y+2z=2$ and $2x+3y-z+1=0$. d) Find an equation for the plane through the points $A(-2,1,4)$, $B(1,0,3)$ that is perpendicular to the plane $4x-y+3z=-1$. c) Let L be the line defined by the parametric equations $x=1-2t$, $y=2+3t$, $z=3+t$. Let $\mathcal{P}$ be the plane defined by $2x+y-z=4$. c1) Show that L and $\mathcal{P}$ are not perpendicular to each other. c2) Find an equation for the plane $\mathcal{Q}$ that both contains L and is perpendicular to $\mathcal{P}$.

9. a) Show that the two lines $L_1: x=1-t,\quad y=2+t,\quad z=1+5t$, and $L_2: x=2+t,\quad y=2+3t,\quad z=-1+7t$ intersect, and find their point of intersection A. b) Find the acute angle θ between L_1 and L_2 at A. c) Find an equation for the plane that contains both L_1 and L_2. d) Find an equation for the plane that contains both L_1 and the point $B(1,-2,-1)$.

10. Find an equation for the surface that results when the elliptic cone $4x^2+9y^2-25z^2=0$ is reflected about the plane: i) $x=0$, ii) $y=0$, iii) $z=0$, iv) $x=y$, v) $y=z$, vi) $z=x$.

11. Show that the two lines $L_1: x=4-t,\quad y=6,\quad z=7+2t$, and $L_2: x=1+7t,\quad y=3+t,\quad z=5-3$ are skew, and find the distance between them.

12. a) Find an equation for the plane $\mathcal{P}$ that contains the line $L: x=3t,\quad y=1+t,\quad z=2t$, and is parallel to the intersection of the planes $y+z=-1$ and $2x-y+z=6$. b) Show that the lines $L_1: x=-2+t,\quad y=3+2t,\quad z=4-t$ and $L_2: x=3-t,\quad y=4-2t,\quad z=t$ are parallel, and find an equation for the plane they determine. c) Find the distance between L_1 and L_2.

13. a) Convert from rectangular to cylindrical coordinates: i) $(4\sqrt{3},4,-4)$, ii) $(-3,3,-1)$.
b) Convert from cylindrical to rectangular coordinates: i) $(4,\frac{\pi}{6},-2)$, ii) $(7\frac{2\pi}{3},5)$.
c) Convert from rectangular to spherical coordinates: i) $(\sqrt{3},1,-2)$, ii) $(-1,1,\sqrt{2})$.
d) Convert from spherical to rectangular coordinates: i) $(3,\frac{5\pi}{6},\frac{4\pi}{3})$, ii) $(4,\frac{7\pi}{12},\frac{\pi}{6})$
e) Convert from cylindrical to spherical coordinates: i) $(\sqrt{5},\frac{3\pi}{4},-3)$, ii) $(3,\frac{11\pi}{6},-2\sqrt{3})$.
f) Convert from spherical to cylindrical coordinates: i) $(5,\frac{\pi}{4},\frac{5\pi}{6})$, ii) $(4,\frac{\pi}{6},\frac{\pi}{2})$.

14. Convert the given equation from a) cylindrical to rectangular coordinates: i) $r=4\sin\theta$, ii) $r=z$, iii) $r^2\cos(2\theta)=z$ b) spherical to rectangular coordinates: i) $\theta=\frac{\pi}{3}$, ii) $\phi=\frac{\pi}{4}$, iii) $\rho=2\sec\phi$, iv) $\rho\sin\phi=2\cos\theta$, v) $\rho=4\cos\phi$, vi) $\rho\sin\phi=\cot\phi$. c) Identify each surface.

15. Describe the domain of the function f in words. a) $f(x,y,z)=\ln(z^2-x^2-y^2)$, b) $f(x,y,z)=\cos^{-1}(x^2+y^2+z^2)$.

16. Sketch the largest region where f is continuous. a) $f(x,y)=\sqrt{x^2+y^2-4}$, b) $f(x,y)=\sin^{-1}(y-x)$.

17. a) Find an equation for the level curve of the function f that passes through the point P. i) $f(x,y)=\int_x^y \frac{dt}{t^2+1}$, $P(-\sqrt{3},\sqrt{3})$. ii) $f(x,y)=\sum_{n=0}^{\infty}(x/y)^n$, $P(1,2)$. b) Find an equation for the level surface of the function f that passes through the point P. i) $f(x,y,z)=\sum_{n=1}^{\infty}\frac{(-1)^n(xyz)^n}{n}$, $P(\sqrt{2},1,1/\sqrt{2})$. ii) $f(x,y,z)=\int_x^y \frac{d\theta}{\sqrt{1-\theta^2}}+\int_{\sqrt{2}}^z \frac{dt}{t\sqrt{t^2-1}}$, $P(0,1/2,2)$. c) Identify the level surfaces of $f(x,y,z)=\ln(x^2+y^2+z^2)$ for $k=-1,\ 0,\ 1$.

18. a) Let $f(x,y,z)=x^2y^3\sin(x^3z^2)$. i) Find $f(y,z,x)$ and $f(z,x,y)$. ii) Find $f_x(x,y,z)$, $f_y(x,y,z)$ and $f_z(x,y,z)$.
b) Use implicit partial differentiation to find $\partial x/\partial y$ and $\partial x/\partial z$ if $xz+y\ln x-x^2+4=0$ defines x as a function of y and z. c) If $x=v\ln u$, $y=u\ln v$, use implicit partial differentiation to find u_x, v_x, u_y and v_y. If we set $z=\tan(2u-3v)$, use the chain rule to find z_x and z_y. d) answer the same questions as in c) if $x=u^2-v^2$, $y=u^2-v$, and $z=u^2+v^2$.

19. Evaluate each limit.

a) $\displaystyle\lim_{(x,y,z)\to(-1,2,1)}\frac{xz^2}{\sqrt{x^2+2y^2+3z^2}}$, b) $\displaystyle\lim_{(x,y)\to(1,1)}\frac{x^2-2xy+y^2}{x^2-y^2}$, c) $\displaystyle\lim_{(x,y)\to(-1,1)}\frac{2x^3+3x^2y-2xy^2-3y^3}{2x^2+xy-y^2}$,

d) $\displaystyle\lim_{(x,y,z)\to(0,0,0)}\frac{xyz}{x^2+y^2+z^2}$, e) $\displaystyle\lim_{(x,y,z)\to(2,2,1)}\frac{\sin(2x-5y+6z)}{(2x-5y+6z)(y+z)}$, f) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{1-\cos(x^2+y^2)}{(x^2+y^2)^2}$.

20. Let $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}-2x+3y, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0).\end{cases}$
a) Find $f_x(0,0)$, and $f_y(0,0)$. b) Show that f is not continuous at (0,0). c) Is f differentiable at $(0,0)$?

21. a) Write down the definition of " f is differentiable at (x_0,y_0)". b) Use the definition in a) to show that the function f given by $f(x,y)=2x-3xy$ is differentiable at the point $(1,-2)$.

22. a) Let $f(x,y,z)=x^3e^{yz}$. i) Find the differential df. ii) Find the local linear approximation for f about $P(1,-1,-1)$, and use it to approximate $f(Q)$ with $Q(0.99,-1.01,-0.98)$. b) Answer the same questions for i) $f(x,y,z)=yz\ln(xy)$, $P(e,1,1)$ and $Q(2.72,0.99,1.01)$. ii) $f(x,y,z)=\tan^{-1}(xyz)$, $P(1,1,1)$ and $Q(0.98,1.01,0.99)$