MAC 2313 (Calculus III)

Test 1 Review. Test 1 will cover all of chapter 11, except for section 11.7. You may skip problem 10 below.

1. Describe the given surface; if it is a sphere, state its radius and center. If it is a point, state its coordinates. a) $x^2+y^2+z^2+6x-2y-6=0$. b) $x^2+y^2+z^2-2mx-6y-8z+50=0$, where m is a parameter, (discuss according to the values of m).

2. a) Find an equation for the sphere passing through the origin and centered at the point $C(1,-2,5)$. b) Decide whether the points $A(2,3,1)$, $B(-1,1,-2)$ and $C(1,-1,1)$ are the vertices of an equilateral triangle.

3. Let $\vec{r}=2\vec{i}-3\vec{j}+4\vec{k}$, $\vec{z}=3\vec{j}-5\vec{k}$, and $\vec{v}=-2\vec{i}+\vec{j}-4\vec{k}$. a) Find the area of the parallelogram having $\vec{r}$ and $\vec{z}$ as adjacent sides. b) Find the volume of the parallelepiped having $\vec{r}$, $\vec{z}$ and $\vec{v}$ as adjacent edges. c) Find the acute angle θ between $\vec{v}$ and the plane containing the face determined by $\vec{r}$ and $\vec{z}$.

4. Consider the lines: $L_1: x=4-2t,\quad y=2+3t,\quad z=1+t$ and $L_2: x=2+4t,\quad y=3-6t,\quad z=-2t$. a) Show that L_1 and L_2 are parallel lines. b) Find the distance between L_1 and L_2.

5. a) Let $A(x_0,y_0,z_0)$ be a given point in 3-space. Let $\mathcal{P}$ be the plane with equation $ax+by+cz+d=0$. Write down the distance D between A and the plane $\mathcal{P}$. $D=$
b) Use a) to find the distance between the two skew lines: $L_1: x=-2+t,\quad y=3+2t,\quad z=1+8t$ and $L_2: x=1-2t,\quad y=-2+3t,\quad z=-1+5t$.

6. Let $\vec{w}=\vec{i}-2\vec{j}+3\vec{k}$ and $\vec{v}=2\vec{i}-\vec{j}-5\vec{k}$. a) Find the vector component of $\vec{v}$ that is parallel to $\vec{w}$ and the vector component of $\vec{v}$ that is orthogonal to $\vec{w}$. b) If θ denotes the angle between $\vec{v}$ and $\vec{w}$, find $\cos(\theta)$ and $\sin(\theta)$. Is θ acute or obtuse? c) Find the direction angles of $\vec{w}$.

7. a) Set $\vec{u}=\vec{i}-3\vec{k}$, $\vec{v}=-\vec{j}+\vec{k}$ and $\vec{w}=2\vec{i}-\vec{j}$. Let $\vec{z}=\vec{i}-\vec{j}+2\vec{k}$. Find scalars a, b, and c such that $\vec{z}=a\vec{u}+b\vec{v}+c\vec{w}$. b) If we now set: $\vec{u}=\vec{i}+\vec{j}-2\vec{k}$, $\vec{v}=\vec{i}+\vec{j}+\vec{k}$ and $\vec{w}=\vec{i}-\vec{j}$, find scalars α, β and γ such that $\vec{z}=\alpha\vec{u}+\beta\vec{v}+\gamma\vec{w}$.

8. a) Find parametric equations for the line through the points $A(-1,2,3)$ and $B(2,-3,4)$. b) Find the vector $\vec{w}$ of norm 4 that is oppositely directed to $\vec{z}=2\vec{i}-\vec{j}+3\vec{k}$. c) Find parametric equations for the line through the point $A(5,0,-2)$ that is parallel to the planes $x-4y+2z=2$ and $2x+3y-z+1=0$. d) Find an equation for the plane through the points $A(-2,1,4)$, $B(1,0,3)$ that is perpendicular to the plane $4x-y+3z=-1$. c) Let L be the line defined by the parametric equations $x=1-2t$, $y=2+3t$, $z=3+t$. Let $\mathcal{P}$ be the plane defined by $2x+y-z=4$. c1) Show that L and $\mathcal{P}$ are not perpendicular to each other. c2) Find an equation for the plane $\mathcal{Q}$ that both contains L and is perpendicular to $\mathcal{P}$.

9. a) Show that the two lines $L_1: x=1-t,\quad y=2+t,\quad z=1+5t$, and $L_2: x=2+t,\quad y=2+3t,\quad z=-1+7t$ intersect, and find their point of intersection A. b) Find the acute angle θ between L_1 and L_2 at A. c) Find an equation for the plane that contains both L_1 and L_2. d) Find an equation for the plane that contains both L_1 and the point $B(1,-2,-1)$.

10. Find an equation for the surface that results when the elliptic cone $4x^2+9y^2-25z^2=0$ is reflected about the plane: i) $x=0$, ii) $y=0$, iii) $z=0$, iv) $x=y$, v) $y=z$, vi) $z=x$.

11. a) If a bug walks on the sphere $x^2+y^2+z^2+2x-2y-4z-3=0$, how close and how far can it get to the origin? b) The distance between the point $P(x,y,z)$ and the point $A(1,-2,0)$ is twice the distance between P and the point $B(0,1,1)$. Show that the set of all such points is a sphere, and find its center and radius.

12. a) Find an equation for the plane $\mathcal{P}$ that contains the line $L: x=3t,\quad y=1+t,\quad z=2t$, and is parallel to the intersection of the planes $y+z=-1$ and $2x-y+z=6$. b) Show that the lines $L_1: x=-2+t,\quad y=3+2t,\quad z=4-t$ and $L_2: x=3-t,\quad y=4-2t,\quad z=t$ are parallel, and find an equation for the plane they determine. c) Find the distance between L_1 and L_2.

13. a) Convert from rectangular to cylindrical coordinates: i) $(4\sqrt{3},4,-4)$, ii) $(-3,3,-1)$.
b) Convert from cylindrical to rectangular coordinates: i) $(4,\frac{\pi}{6},-2)$, ii) $(7\frac{2\pi}{3},5)$.
c) Convert from rectangular to spherical coordinates: i) $(\sqrt{3},1,-2)$, ii) $(-1,1,\sqrt{2})$.
d) Convert from spherical to rectangular coordinates: i) $(3,\frac{5\pi}{6},\frac{4\pi}{3})$, ii) $(4,\frac{7\pi}{12},\frac{\pi}{6})$
e) Convert from cylindrical to spherical coordinates: i) $(\sqrt{5},\frac{3\pi}{4},-3)$, ii) $(3,\frac{11\pi}{6},-2\sqrt{3})$.
f) Convert from spherical to cylindrical coordinates: i) $(5,\frac{\pi}{4},\frac{5\pi}{6})$, ii) $(4,\frac{\pi}{6},\frac{\pi}{2})$.

14. Convert the given equation from a) cylindrical to rectangular coordinates: i) $r=4\sin\theta$, ii) $r=z$, iii) $r^2\cos(2\theta)=z$ b) spherical to rectangular coordinates: i) $\theta=\frac{\pi}{3}$, ii) $\phi=\frac{\pi}{4}$, iii) $\rho=2\sec\phi$, iv) $\rho\sin\phi=2\cos\theta$, v) $\rho=4\cos\phi$, vi) $\rho\sin\phi=\cot\phi$. c) Identify each surface.

15. Review all the true/false problems from chapter 11 in the text.