

MAC 2313 (Calculus III)
Test 1 Review

1. Describe the given surface; if it is a sphere, state its radius and center. If it is a point, state its coordinates. a) $x^2 + y^2 + z^2 + 6x - 2y - 6 = 0$. b) $x^2 + y^2 + z^2 - 2mx - 6y - 8z + 50 = 0$, where m is a parameter. (discuss according to the values of m .)
2. a) Find an equation for the sphere passing through the origin and centered at the point $C(1, -2, 5)$. b) Decide whether the points $A(2, 3, 1)$, $B(-1, 1, -2)$ and $C(1, -1, 1)$ are the vertices of an equilateral triangle.
3. Let $\vec{r} = 2\vec{i} - 3\vec{j} + 4\vec{k}$, $\vec{z} = 3\vec{j} - 5\vec{k}$, and $\vec{v} = -2\vec{i} + \vec{j} - 4\vec{k}$. a) Find the area of the parallelogram having \vec{r} and \vec{z} as adjacent sides. b) Find the volume of the parallelepiped having \vec{r} , \vec{z} and \vec{v} as adjacent edges. c) Find the acute angle θ between \vec{v} and the plane containing the face determined by \vec{r} and \vec{z} .
4. Consider the lines: $L_1 : x = 4 - 2t, y = 2 + 3t, z = 1 + t$ and $L_2 : x = 2 + 4t, y = 3 - 6t, z = -2t$. a) Show that L_1 and L_2 are parallel lines. b) Find the distance between L_1 and L_2 .
5. a) Let $A(x_0, y_0, z_0)$ be a given point in 3-space. Let \mathcal{P} be the plane with equation $ax + by + cz + d = 0$. Write down the distance D between A and the plane \mathcal{P} . $D =$
b) Use a) to find the distance between the two skew lines: $L_1 : x = -2 + t, y = 3 + 2t, z = 1 + 8t$ and $L_2 : x = 1 - 2t, y = -2 + 3t, z = -1 + 5t$.
6. Let $\vec{w} = \vec{i} - 2\vec{j} + 3\vec{k}$ and $\vec{v} = 2\vec{i} - \vec{j} - \vec{k}$. Find the vector component of \vec{v} that is parallel to \vec{w} and the vector component of \vec{v} that is orthogonal to \vec{w} .
7. a) Set $\vec{u} = \vec{i} - 3\vec{k}$, $\vec{v} = -\vec{j} + \vec{k}$ and $\vec{w} = 2\vec{i} - \vec{j}$. Let $\vec{z} = \vec{i} - \vec{j} + 2\vec{k}$. Find scalars a, b , and c such that $\vec{z} = a\vec{u} + b\vec{v} + c\vec{w}$. b) If we now set: $\vec{u} = \vec{i} + \vec{j} - 2\vec{k}$, $\vec{v} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{w} = \vec{i} - \vec{j}$, find scalars α, β and γ such that $\vec{z} = \alpha\vec{u} + \beta\vec{v} + \gamma\vec{w}$.
8. a) Find parametric equations for the line through the points $A(-1, 2, 3)$ and $B(2, -3, 4)$. b) Find the vector \vec{w} of norm 4 that is oppositely directed to $\vec{z} = 2\vec{i} - \vec{j} + 3\vec{k}$. c) Find parametric equations for the line through the point $A(5, 0, -2)$ that is parallel to the planes $x - 4y + 2z = 2$ and $2x + 3y - z + 1 = 0$. d) Find an equation for the plane through the points $A(-2, 1, 4)$, $B(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = -1$. c) Let L be the line defined by the parametric equations $x = 1 - 2t, y = 2 + 3t, z = 3 + t$. Let \mathcal{P} be the plane defined by $2x + y - z = 4$. c1) Show that L and \mathcal{P} are not perpendicular to each other. c2) Find an equation for the plane \mathcal{Q} that both contains L and is perpendicular to \mathcal{P} .
9. a) Show that the two lines $L_1 : x = 1 - t, y = 2 + t, z = 1 + 5t$, and $L_2 : x = 2 + t, y = 2 + 3t, z = -1 + 7t$ intersect, and find their point of intersection A . b) Find the acute angle θ between L_1 and L_2 at A . c) Find an equation for the plane that contains both L_1 and L_2 . d) Find an equation for the plane that contains both L_1 and the point $B(1, -2, -1)$.
10. Find an equation for the surface that results when the elliptic cone $4x^2 + 9y^2 - 25z^2 = 0$ is reflected about the plane: i) $x = 0$, ii) $y = 0$, iii) $z = 0$, iv) $x = y$, v) $y = z$, vi) $z = x$.
11. Show that the two lines $L_1 : x = 4 - t, y = 6, z = 7 + 2t$, and $L_2 : x = 1 + 7t, y = 3 + t, z = 5 - 3t$ are skew, and find the distance between them.
12. a) Find an equation for the plane \mathcal{P} that contains the line $L : x = 3t, y = 1 + t, z = 2t$, and is parallel to the intersection of the planes $y + z = -1$ and $2x - y + z = 6$. b) Show that the lines $L_1 : x = -2 + t, y = 3 + 2t, z = 4 - t$ and $L_2 : x = 3 - t, y = 4 - 2t, z = t$ are parallel, and find an equation for the plane they determine. c) Find the distance between L_1 and L_2 .
13. a) Convert from rectangular to cylindrical coordinates: i) $(4\sqrt{3}, 4, -4)$, ii) $(-3, 3, -1)$.
b) Convert from cylindrical to rectangular coordinates: i) $(4, \frac{\pi}{6}, -2)$, ii) $(7\frac{2\pi}{3}, 5)$.
c) Convert from rectangular to spherical coordinates: i) $(\sqrt{3}, 1, -2)$, ii) $(-1, 1, \sqrt{2})$.
d) Convert from spherical to rectangular coordinates: i) $(3, \frac{5\pi}{6}, \frac{4\pi}{3})$, ii) $(4, \frac{7\pi}{12}, \frac{\pi}{6})$.
e) Convert from cylindrical to spherical coordinates: i) $(\sqrt{5}, \frac{3\pi}{4}, -3)$, ii) $(3, \frac{11\pi}{6}, -2\sqrt{3})$.
f) Convert from spherical to cylindrical coordinates: i) $(5, \frac{\pi}{4}, \frac{5\pi}{6})$, ii) $(4, \frac{\pi}{6}, \frac{\pi}{2})$.
14. Convert the given equation from a) cylindrical to rectangular coordinates: i) $r = 4 \sin \theta$, ii) $r = z$, iii) $r^2 \cos(2\theta) = z$
b) spherical to rectangular coordinates: i) $\theta = \frac{\pi}{3}$, ii) $\phi = \frac{\pi}{4}$, iii) $\rho = 2 \sec \phi$, iv) $\rho \sin \phi = 2 \cos \theta$, v) $\rho = 4 \cos \phi$, vi) $\rho \sin \phi = \cot \phi$. c) Identify each surface.