MAC 2311 (Calculus I)
Test 2, Monday November 21, 2011

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University administration. Good Luck.

1. [10] Find the following limits.

a) $\lim_{x \to \frac{\pi}{2}^+} \sec(5x)\cos(7x) =$

b) $\lim_{x \to 0} (1 - 3x)^{\frac{1}{2x}} =$

2. [20] Let $f(x) = x^3 + 6x^2 + 9x + 2$. a) Find the first and second derivatives of f. b) Find the intervals of increase and decrease. c) Find the intervals of concavity and the inflection point(s).

3. [20] Let $f(x) = x^2 e^x$. Find all the critical points of f and state for each critical point whether a relative maximum, a relative minimum, or neither occurs. (You must show all your work, and clearly state which theorem you are using for the classification.)

4. [10] Find the maximum rectangular area that can be fenced with \$2400 if two opposite sides of the rectangle will use fencing costing \$4 per foot and the remaining sides will use fencing costing \$6 per foot.

5. [10] Decide whether the statement is true or false. No explanation is needed.

a) If f has a relative minimum at $x = 1.01$, then $f(1.01) \leq f(1)$.

b) $\int f(x)g(x)\,dx = \int f(x)\,dx \int g(x)\,dx$.

c) If f has a relative extremum at $x = -2$, then $x = -2$ is a critical point of f.

d) If f and g have the same derivative function on an interval I, then f and g differ by a constant on I.

e) If $f'(x_0) = 0$ and $f''(x_0) > 0$, then f has a local maximum at x_0.

6. [20] Find the following indefinite integrals.

a) $\int (x^{25} - \frac{10}{\sqrt[5]{x}} + \frac{7}{x^3})\,dx =$

b) $\int \frac{\sqrt{1-x^2}}{-x^2+1}\,dx =$

c) $\int (\sin x - 3\sec x \tan x\,dx =$

d) $\int \frac{x^3+x-2}{1+x^2}\,dx =$

7. [10] a) State the Mean Value Theorem. b) Show that the function f defined by $f(x) = \sqrt{x} - 2x, \quad x$ in $[0, 1]$, satisfies all the requirements of the Mean Value Theorem. c) Find all numbers x_0 in $(0, 1)$ such that $f'(x_0) = \frac{f(1)-f(0)}{1-0}$.