MAC 2312 (Calculus II)
Test 2, Wednesday April 11, 2012

Name: **PID:**

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; no credits will be awarded for unexplained answers. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [10] Use division to find the first three nonzero terms of the Maclaurin series for $f(x) = \dfrac{e^x}{\cos x}$.

2. [10] Find the Taylor polynomial of order four for $f(x) = \ln x$ about $x = 2$.

3. [10] Decide whether each statement is true or false.

a) If the series $\sum |u_k|$ converges, then the series $\sum u_k$ converges.

b) If the series $\sum u_k$ converges, then $\lim_{k\to\infty} u_k = 0$.

c) If $\lim_{k\to\infty} \sqrt[k]{|u_k|} = 1$, then the series $\sum u_k$ converges conditionally.

d) Every alternating series converges.

e) If $0 < a_k \leq b_k$ for all $k \geq 1$, and $\sum a_k$ converges, then $\sum b_k$ converges too.

4. [10] a) Use a popular Maclaurin series to find the Maclaurin series for $f(x) = \sin(x^2)$. b) Use the Maclaurin series obtained in a) to evaluate the integral $\int_0^{\pi} \sin(x^2)\, dx$.

5. [8] Sketch the region enclosed by the curves $y = 2x$, $y = 4 - x$, $y = 0$, and find its area.

6. [12] a) Use cylindrical shells to find the volume of the solid generated when the region enclosed by the curves $y = 1/\sqrt{x^2+1}$, $x = 0$, $x = 1$, $y = 0$, is revolved about the y-axis.

b) Find the area of the surface that results when the curve $y = \sqrt{x}$, $1 \leq x \leq 2$, is revolved about the x-axis.

7. [10] Determine the radius of convergence and the interval of convergence of the power series $\sum_{k=1}^{\infty} \frac{(-1)^k (x+5)^k}{3^k \sqrt{k}}$.

8. [8] Use the integral test to decide whether the infinite series $\sum_{k=1}^{\infty} \frac{k}{k^2+3}$ converges or diverges.

9. [6] Find an upper bound on the absolute error if the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt[4]{k}}$ is approximated by its n^{th} partial sum s_n with $n = 9999$.

10. [8] Find the exact length of the arc of the curve $x = \sqrt{1-y^2}$, $0 \leq y \leq 1/2$.

11. [8] A cylindrical tank of radius 4 ft and height 12ft is to be filled to five-sixth with water. Find the work required to pump the water in through an opening located at the bottom of the tank. (Remember that the weight density of water is 62.4lb/ft^3.)