MAC 2311 (Calculus I)
Test 2 , Wednesday February 25, 2015 — Answers

Name: PID:

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you must show all your work to deserve the full mark assigned to any question. Guessing the correct answers won't earn you any credits. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University administration. 2 pages. Total=62 points. Good Luck!

1. [30] Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit(s) by guessing the correct answer(s).)

a) $f(x) = \frac{3x}{x^2-5x+7}$

$$f'(x) = \frac{3(x^2-5x+7)-(2x-5)(3x)}{(x^2-5x+7)^2} = \frac{3x^2-15x+21-6x^2+15x}{(x^2-5x+7)^2} = \frac{-3x^2+21}{(x^2-5x+7)^2}$$

b) $g(x) = x^4\cos(1+x^2)$

$$g'(x) = 4x^3\cos(1+x^2) - 2x\cdot x^4\sin(1+x^2) = 2x^3(2\cos(1+x^2) - x^2\sin(1+x^2))$$

c) $h(x) = \sin^{-1}(2^x)$

$$h'(x) = \frac{(\ln 2)2^x}{\sqrt{1-2^{2x}}}$$

Note: $(2^x)^2 = 2^{2x}$.

d) $k(x) = 5x^3 - \frac{6}{\sqrt[5]{x}} + \frac{3}{x^2} - \pi^4 = 5x^3 - 6x^{-\frac{1}{5}} + 3x^{-2} - \pi^4$

$$k'(x) = 15x^2 + \frac{6}{5}x^{-\frac{6}{5}} - 6x^{-3} = 15x^2 + \frac{6}{5\sqrt[5]{x^6}} - \frac{6}{x^3}$$

e) Use the logarithmic differentiation technique to find $\frac{dy}{dx}$ if $y = (x-e^x)^{\tan x}$.

$$\ln y = \ln[(x-e^x)^{\tan x}] = \tan x \ln(x-e^x)$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\tan x \ln(x-e^x)) = (\sec^2 x)\ln(x-e^x) + \frac{(\tan x)(1-e^x)}{x-e^x}$$

$$\frac{dy/dx}{y} = (\sec^2 x)\ln(x-e^x) + \frac{(\tan x)(1-e^x)}{x-e^x}; \text{ so}$$

$$\frac{dy}{dx} = \left[(\sec^2 x)\ln(x-e^x) + \frac{(\tan x)(1-e^x)}{x-e^x}\right](x-e^x)^{\tan x}$$

f) Use the implicit differentiation technique to find $\frac{dy}{dx}$ if $x^2y + 2y + 2x\tan(y) = 2$.

$$\frac{d}{dx}(x^2y + 2y + 2x\tan y) = \frac{d}{dx}(2) = 0$$

$$2xy + x^2\frac{dy}{dx} + 2\frac{dy}{dx} + 2\tan y + 2x(\sec^2 y)\frac{dy}{dx} = 0$$

$$(x^2 + 2 + 2x\sec^2 y)\frac{dy}{dx} = -2xy - 2\tan y; \text{ so } \frac{dy}{dx} = \frac{-2xy-2\tan y}{x^2+2+2x\sec^2 y}$$

2. [6] Find all values of x at which the line that is tangent to $y = \frac{x^3}{3} - \frac{3}{2}x^2 + x + 2$ is perpendicular to the line $y - x = 2$.

Slope of tangent line $= \frac{dy}{dx} = x^2 - 3x + 1$. We need $\frac{dy}{dx} = -1$; this gives $x^2 - 3x + 1 = -1$ or $x^2 - 3x + 2 = 0$ or $(x-1)(x-2) = 0$

so $x = 1$ or $x = 2$.

3. [6] Write down the correct statement.

a) Constant rule: $\frac{d}{dx}(c) = 0$, c = constant

b) Power rule: $\frac{d}{dx}(x^r) = r x^{r-1}$, r = real number

c) Product rule: $(fg)'(x) = f'(x)g(x) + g'(x)f(x)$

d) Quotient rule: $(f/g)'(x) = \dfrac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$

e) Chain rule: $(f(g))'(x) = g'(x)f'(g(x))$

f) Write down the definition: $f'(x_0) = \lim_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$ or $f'(x_0) = \lim_{h \to 0} \dfrac{f(x_0+h) - f(x_0)}{h}$

4. [10] Let $y = x^3 + 2$. a) Find the average rate of change of y with respect to x on the interval $[-2, 1]$. b) Find the instantaneous rate of change of y with respect to x at $x_0 = 1$, and use it to find the tangent line to the curve $y = x^3 + 2$ at $x = 1$.

a) $r_{ave} = \dfrac{f(1) - f(-2)}{1 - (-2)} = \dfrac{3 - (-8+2)}{3} = \dfrac{3+6}{3} = 3$

b) $r_{inst} = \lim_{x_1 \to 1} \dfrac{f(x_1) - f(1)}{x_1 - 1} = \lim_{x_1 \to 1} \dfrac{x_1^3 + 2 - 3}{x_1 - 1} = \lim_{x_1 \to 1} \dfrac{x_1^3 - 1}{x_1 - 1}$

$= \lim_{x_1 \to 1} \dfrac{\cancel{(x_1 - 1)}(x_1^2 + x_1 + 1)}{\cancel{x_1 - 1}}$

$= \lim_{x_1 \to 1} (x_1^2 + x_1 + 1)$

$= 3$

Equation of tangent line:

$y = 3 + 3(x-1)$

or $y = 3x$.

5. [10] Mark each statement true or false. No explanation needed.

a) If $k(x) = \cos^2(x^2) + \sin^2(x^2)$, then $k'(x) = 0$. True, as $k(x) = 1$ for all x.

b) If $g(x) = e^x$, then $g'(x) = e^x$. True

c) If $\lim_{h \to 0} \dfrac{f(-4+h) - f(-4)}{h} = 25$, then $\lim_{x \to -4} \dfrac{f(x) - f(-4)}{x+4} = 25$. True, see definition of the derivative.

d) If $f(x) = \frac{\sin x}{g(x)}$, then $f'(x) = \frac{\cos x}{g'(x)}$. false, see the quotient rule.

e) $\frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)}$. false, see logarithmic differentiation.

f) If $f(x) = h(\cos x)$, then $f'(x) = h'(-\sin x)$. False, see the chain rule.

g) If f is differentiable at $x = 3$, then f is continuous at $x = 3$. True, by Theorem 2-2-3 in text.

h) If $\lim_{h \to 0} \dfrac{f(2+h) - f(2)}{h}$ exists, then f is differentiable at $x = 2$. True, by Definition 2.2.2 in text

i) If $p(x) = f(x)\cot x$, then $p'(x) = -f'(x)\csc^2 x$. False, see the product rule.

j) If $m(x) = \ln(4)$, then $m'(x) = 1/4$. False, see the constant rule.