MAC 2312 (Calculus II)
Test 2, Wednesday October 07, 2013

Name: PID:

Remember that no documents or calculators are allowed during the test. Be as precise as possible in your work; guessing the correct answer won't earn you any credit. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University Administration. Good luck.

1. [10] Determine whether the improper integral converges or diverges. If it converges, state its limit, and if it diverges, state whether it diverges to $+\infty$ or to $-\infty$.

a) $\int_0^1 \frac{1}{x^{\frac{7}{8}}}\,dx = \lim_{r\to 0^+} \int_r^1 x^{-\frac{7}{8}}dx$

$= \lim_{r\to 0^+} 8x^{\frac{1}{8}}\Big]_r^1$

$= 8(1 - \lim_{r\to 0^+} r^{\frac{1}{8}})$

$= 8$

So I_i converges to 8

b) $\int_3^{+\infty} \frac{dx}{x(\ln x)^{\frac{3}{2}}} = \lim_{R\to+\infty} \int_3^R \frac{dx}{x(\ln x)^{3/2}}$

$u = \ln x$
$du = \frac{dx}{x}$

$= \lim_{R\to+\infty} \int_{\ln 3}^{\ln R} \frac{du}{u^{3/2}}$

$= \lim_{R\to+\infty} -\frac{2}{\sqrt{u}}\Big]_{\ln 3}^{\ln R}$

$= \frac{2}{\sqrt{\ln 3}} - 2\lim_{R\to+\infty} \frac{1}{\sqrt{\ln R}}$

$= \frac{2}{\sqrt{\ln 3}} - 0 = \frac{2}{\sqrt{\ln 3}}$

So I_i converges to $\frac{2}{\sqrt{\ln 3}}$

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2. [12] a) Write down the partial fractions decomposition of the rational function

$R(x) = \frac{2x^3-5x^2+7x-11}{x(x-3)^3(x^2+2)(x^2+1)^2} = \frac{a_1}{x} + \frac{b_1}{x-3} + \frac{b_2}{(x-3)^2} + \frac{c_1x+d_1}{x^2+2} + \frac{c_2x+d_2}{x^2+1} + \frac{c_3x+d_3}{(x^2+1)^2}$

b) Evaluate the integral $\int \frac{2x+7}{(x+2)(x^2+1)}dx = \int \frac{a_1}{x+2} + \frac{b_1x+c_1}{x^2+1}\,dx$, a_1, b_1, c_1 are real numbers

$= a_1 \ln|x+2| + \frac{b_1}{2}\ln(x^2+1) + c_1 \tan^{-1}(x) + C$

For a_1: multiply both sides of $\frac{2x+7}{(x+2)(x^2+1)} = \frac{a_1}{x+2} + \frac{b_1x+c_1}{x^2+1}$ by $x+2$, simplify

and set $x=-2$: $a_1 = \frac{-4+7}{4+1} = \frac{3}{5}$

For b_1, multiply both sides by x^2+1, simplify, and set $x = +i$

$b_1 i + c_1 = \frac{2i+7}{i+2} = \frac{(2i+7)(i-2)}{(i+2)(i-2)} = \frac{2i^2-4i+7i-14}{i^2-4} = \frac{-16+3i}{-5}$

Hence $b_1 = -\frac{3}{5}$, $c_1 = \frac{16}{5}$.

3. [8] Approximate the integral $\int_0^{\pi^2} \sin(\sqrt{x})\,dx$ using: a) the trapezoidal rule with $n = 2$. b) Simpson's rule with $n = 2$. c) the midpoint rule with $n = 2$.

$x_0 = 0, \quad x_1 = \frac{\pi^2}{2}, \quad x_2 = \pi^2; \quad x_{0,1} = \frac{\pi^2}{4}, \quad x_{1,2} = \frac{3\pi^2}{4}$

a) $\int_0^{\pi^2} \sin(\sqrt{x})dx \approx T_2 = \frac{\pi^2}{4}(\sin 0 + 2\sin(\frac{\pi}{\sqrt{2}}) + \sin\pi) = \frac{\pi^2}{2}\sin(\frac{\pi}{\sqrt{2}})$

b) $\int_0^{\pi^2} \sin(\sqrt{x})dx \approx S_2 = \frac{\pi^2}{6}(\sin 0 + 4\sin(\frac{\pi}{\sqrt{2}}) + \sin\pi) = \frac{2\pi^2}{3}\sin(\frac{\pi}{\sqrt{2}})$

c) $\int_0^{\pi^2} \sin(\sqrt{x})dx \approx M_2 = \frac{\pi^2}{2}(\sin\frac{\pi}{2} + \sin(\frac{\sqrt{3}\pi}{2})) = \frac{\pi^2}{2}(1 + \sin(\frac{\sqrt{3}\pi}{2}))$

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4. [10] Integrate by parts. (Show all your work. No credits if no work is shown.)

a) $\int_1^4 \frac{\ln x}{\sqrt{x}}\,dx = 2\sqrt{x}\ln x\Big]_1^4 - \int_1^4 \frac{2\sqrt{x}}{x}dx$

$u = \ln x; \quad dv = \frac{dx}{\sqrt{x}}$

$du = \frac{dx}{x}; \quad v = 2\sqrt{x}$

$= 4\ln 4 - 2\ln 1 - 4\sqrt{x}\Big]_1^4$

$= 4\ln 4 - 4(2-1)$

$= 4(\ln 4 - 1)$

b) $\int \ln(1 + x^2)\,dx = x\ln(1+x^2) - \int \frac{x \cdot 2x}{1+x^2}dx$

$u = \ln(1+x^2)$

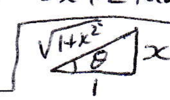
$dv = dx; \quad v = x$

$du = \frac{2x}{1+x^2}dx$

$= x\ln(1+x^2) - 2\int \frac{x^2+1-1}{x^2+1}dx$

$= x\ln(1+x^2) - 2\int 1 - \frac{1}{x^2+1}dx$

$= x\ln(1+x^2) - 2x + 2\tan^{-1}x + C$



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5. [10] Evaluate each indefinite integral. (Show all your work. No credits if no work is shown.)

a) $\int \sin(2x)\cos(4x)\,dx = \int \frac{\sin 6x + \sin(-2x)}{2}dx$

$= -\frac{\cos(6x)}{12} + \frac{\cos(2x)}{4} + C$

Remember that $\sin(-2x) = -\sin(2x)$

b) $\int \frac{dx}{x^4+2x^2+1} = \int \frac{dx}{(x^2+1)^2} = \int \frac{\sec^2\theta\,d\theta}{\sec^4\theta}$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$= \int \frac{d\theta}{\sec^2\theta}$

$= \int \cos^2\theta\,d\theta$

$= \frac{1}{2}\int (1+\cos(2\theta))d\theta$

$= \frac{1}{2}(\theta + \frac{\sin(2\theta)}{2}) + C$

$= \frac{1}{2}(\tan^{-1}x + \frac{x}{1+x^2}) + C,$

since

$\sin\theta = \frac{x}{\sqrt{1+x^2}}$

$\cos\theta = \frac{1}{\sqrt{1+x^2}}$

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6. [10, **Bonus**] Evaluate the integrals (Show all your work. No credits if no work is shown.)

a) $\int_0^{\frac{\pi}{3}} \sqrt{\sec x}\tan x\,dx = \int_0^{\pi/3} \sec^{-1/2}x \sec x \tan x\,dx$

$u = \sec x$

$du = \sec x \tan x\,dx$

$= \int_1^2 u^{-1/2}du$

$= 2\sqrt{u}\Big]_1^2$

$= 2(\sqrt{2} - 1)$

b) $\int \frac{dx}{e^{-x}+1} = \int \frac{dx}{e^{-x}(1+e^x)}$

$= \int \frac{e^x dx}{1+e^x}$

$u = e^x + 1$

$du = e^x dx$

$= \int \frac{du}{u}$

$= \ln|u| + C$

$= \ln(1+e^x) + C$