MAC 2311 (Calculus I)

Test 2 Review- Test 2 is due on 11/03/17, and will cover chap 3 and sections 4.1 and 4.2.

1. Let l be the length of a diagonal in a rectangle whose sides have lengths x and y, and assume that x and y vary with time. a) How are dl/dt, dx/dt and dy/dt related? b) If x increases at a rate of 6 in/s and y decreases at a rate of 3 in/s, how fast is l changing when $x=3$ft and $y=4$ft? Is the diagonal increasing or decreasing at that instant?

2. A 17ft ladder is leaning against a wall. If the bottom of the ladder is pulled along the ground away from the wall at a constant rate of 5ft/s, how fast will the top of the ladder be moving down the wall when it is 8ft above the ground?

3. A point P is moving along the curve $y=\sqrt{x^3+17}$. When P is at (2,5), y is increasing at a rate of 2 units/s. How fast is x changing?

4. An aircraft is climbing at a 30° angle to the horizontal. How fast is the aircraft gaining altitude if its speed is 500 mi/h?

5. A steel cube with 1-inch side is coated with 0.01 inch of copper. a) What is the volume of copper in the coating? b) Use differentials to estimate the volume of copper in the coating.

6. Find the differential dy if: a) $y=\frac{1-x^3}{2-x}$, b) $y=\cos(\sin x)$, c) $y=\tan^{-1}(e^{\sin x})$.

7. Use the differential dy to approximate Δy when x changes as indicated. a) $y=\frac{x}{x^2+1}$; from $x=2$ to $x=1.96$. b) $y=x\sqrt{x^2+3}$; from $x=1$ to $x=1.02$.

8. Use an appropriate local linear approximation to estimate the given value. a) $\sqrt{80.82}$, b) $\ln(1.01)$, c) $\tan(44°)$, d) $\sin(31°)$, e) $\sqrt[3]{9.02}$.

9. Evaluate each limit.

a) $\displaystyle\lim_{x\to 0^+}\frac{1-\ln x}{e^{\frac{1}{x}}}$, b) $\displaystyle\lim_{x\to+\infty}x^2(1-\cos(1/x))$, c) $\displaystyle\lim_{x\to 0}x^2(1-\cos(1/x))$, d) a, b are constants with $b>0$. $\displaystyle\lim_{x\to-\infty}(1+\frac{a}{x})^{bx}$, e) $\displaystyle\lim_{x\to\frac{\pi}{2}^-}(\tan x)^{(\frac{\pi}{2}-x)}$, f) $\displaystyle\lim_{x\to 0}\frac{x-\tan x}{x^3}$, g) $\displaystyle\lim_{x\to 0^-}(\frac{1}{x}-\cot x)$,
h) $\displaystyle\lim_{x\to+\infty}\frac{x}{\sqrt{x^2+1}}$.

10. Mark each statement true or false.

a) If f is decreasing on [0,2], then $f(0)>f(1)>f(2)$.
b) If $f'(1)>0$, then f is increasing on [0,2].
c) If f' is increasing on [0,1] and f' is decreasing on [1,2], then $(1,f(1))$ is an inflection point of f.
d) If f has a relative maximum at $x=1$, then $f(1)\geq f(2)$.
e) If f has a relative maximum at $x=-1$, then $x=-1$ is a critical point of f.
f) If $f''(1)>0$, then f has a relative minimum at $x=1$.
g) If $f'(-2)=0$, then f has a relative maximum or a relative minimum at $x=-2$.
h) If $f'(-3)=0$ and $f''(-3)<0$, then f has a relative maximum at $x=-3$.
i) If $f''(5)=0$, then the point $(5,f(5))$ is an inflection point of f.
j) If $f'(3)$ does not exist, then $x=3$ is a critical point of f.

11. Find the intervals of increase, decrease, concavity, and the inflection points of f.

a) $f(x)=x^{\frac{4}{3}}-x^{\frac{1}{4}}$, b) $f(x)=x^4-5x^3+9x^2$, c) $f(x)=x^3\ln x$, d) $f(x)=xe^{-x^2}$,
e) $f(x)=\tan^{-1}(1-x^2)$.

12. Find and classify all the critical points of f as points of local maximum, local minimum or neither.

a) $f(x)=x^4-12x^3+1$, b) $f(x)=x^2(x+1)^{\frac{2}{3}}$, c) $f(x)=\sqrt{3}x-\sin(2x)$, $0\leq x\leq 2\pi$,

d) $f(x) = x^2 e^{2x}$, e) $f(x) = |4x - x^3|$.

13. Find all the intercepts, asymptotes, intervals of increase, decrease, concavity, critical points, inflection points of f.

a) $f(x) = \frac{x^3-4x-8}{x+2}$, b) $f(x) = x^{\frac{2}{3}} e^x$, c) $f(x) = \sin^2 x - \cos x$, $-\pi \le x \le \pi$.

14. Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit by guessing the correct answer(s).)

a) $f(x) = e^{\ln(x^2+1)}$

b) $g(x) = \frac{2x+\ln x}{3x+\ln x}$

c) $h(x) = x^3 \ln(x^2+2) - 2x(\ln x)^3$

d) $k(x) = 4^{\tan^{-1}(x)}$

e) $l(x) = \sin^{-1}(3x) - 3\tan^{-1}(e^{\cos x})$

f) $m(x) = \sec^{-1}(2x)$

g) Use the logarithmic differentiation technique to find the derivative of $p(x) = (x - e^x)^{\sin x}$.

h) Use the implicit differentiation technique to find $\frac{dy}{dx}$ if $x + y^2 - \sin(xy) = 2$. Find an equation for the tangent line to the curve $x + y^2 - \sin(xy) = 2$ at the point $(2, 0)$.

15. Show that the function $y = e^{2x}\cos(3x)$ solves the equation $y'' - 4y' + 13y = 0$.