MAC 2311 (Calculus I)
Test 2 Review, Spring 2015

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you shall show all your work to deserve the full mark assigned to any question. Guessing the correct answers won't earn you any credits. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University administration. Good Luck!

1. [32] Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit(s) by guessing the correct answer(s).)

a) $f(x) = \frac{2x}{x^2-3x+1}$

b) $g(x) = x^3 \ln(1+x^2)$

c) $h(x) = \tan^{-1}(x^4)$

d) $k(x) = 3^{\cos x} - e^{(-2x^3+5x^2-3x+7)}$

e) Use the logarithmic differentiation technique to find $\frac{dy}{dx}$ if $y = (x+\tan x)^{\sin x}$.

f) Use the implicit differentiation technique to find $\frac{dy}{dx}$ if $xy + x^2\cos(y) = 1$.

g) $p(x) = (2x+7)^9(3x-5)^6$

h) $m(x) = \frac{1+x^2}{\cot x+\csc x}$

i) $n(x) = \log_{2x-4}(5x+3)$

j) Find all values of x at which the line that is tangent to $y = 3x - \tan x$ is parallel to the line $y - x = 2$.

2. [6] Use the definition of the derivative to evaluate the limits

a) $\lim_{x \to 2} \frac{\sec(\pi x/8) - \sqrt{2}}{x - 2} =$

b) $\lim_{x \to 1} \frac{x^{11} - 1}{x - 1} =$

3. [5] Let $y = 2x^2 - 3$. a) Find the average rate of change of y with respect to x on the interval $[-1, 2]$. b) Find the instantaneous rate of change of y with respect to x at $x_0 = -1$.

4. [5] a) Write down the two definitions for $f'(x_0)$. b) Use any of those definitions to find $f'(1)$ if $f(x) = \sqrt{x}$. c) Use part b) to find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 1$.

5. [5] If $f(x) = \begin{cases} 3x^2 - 5, & x > -1 \\ 5x^3 + 3, & x \leq -1. \end{cases}$

a) Show that f is continuous at $x = -1$. b) Is f differentiable at $x = -1$? You must carefully explain your answer to get any credits.

6. [34] Find the derivative of each of the following functions (Show all your work, and simplify your answers as much as possible; you will not get any credit by guessing the correct answer(s).)

a) $f(x) = -4x^3 - \frac{8}{\sqrt[4]{x}} + \frac{7}{x^5}$

b) $g(x) = \frac{4x-5}{x^2+x+1}$

c) $h(x) = x^3 \sin(x^2)$

d) $k(x) = \sec^2(e^{\sin x}) - \tan^2(e^{\sin x})$

e) $l(x) = \sin^{-1}(3x) - 3\tan^{-1}(e^{\cos x})$

f) $m(x) = \cos(\cos x)$

g) Use the logarithmic differentiation technique to find the derivative of $p(x) = (x - e^x)^{\sin x}$.

h) Use the implicit differentiation technique to find $\frac{dy}{dx}$ if $x + y^2 - \sin(xy) = 2$. Find an equation for the tangent line to the curve $x + y^2 - \sin(xy) = 2$ at the point $(2, 0)$.

i) Suppose that a function f is differentiable at $x = 2$, and $\lim_{x \to 2} \dfrac{x^3 f(x) - 24}{x - 2} = 28$. Find $f(2)$ and $f'(2)$.

j) Find all values of x at which the tangent line to the curve $y = 2x^3 - x^2$ is perpendicular to the line $x + 4y = 10$.

7. [10] Decide whether the statement is true or false. No explanation needed.

a) If $f(x) = \frac{\sin x}{g(x)}$, then $f'(x) = \frac{\cos x}{g'(x)}$.

b) $\frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)}$.

c) If $f(x) = h(\cos x)$, then $f'(x) = h'(-\sin x)$.

d) If f is differentiable at -7, then f is continuous at -7.

e) If f is continuous at 2, then f is differentiable at 2.

f) If $g(x) = e^{\cos x}$, then $g'(x) = e^{\cos x}$.

g) If $k(x) = \cos^2(x^2) + \sin^2(x^3)$, then $k'(x) = 0$.

h) If $p(x) = f(x) \tan x$, then $p'(x) = f'(x) \sec^2 x$.

i) If $\lim_{x \to -1} \dfrac{f(x) - f(-1)}{x + 1} = -2$, then $\lim_{h \to 0} \dfrac{f(-1 + h) - f(-1)}{h} = -2$.

j) If $m(x) = e^6$, then $m'(x) = 6e^5$.