# MAC 2313 (Calculus III)
## Test 2 Review: 11.7, 13.1 to 13.7

1. Describe the domain of the function $f$ in words. a) $f(x,y,z) = \ln(z^2 - x^2 - y^2)$, b) $f(x,y,z) = \cos^{-1}(x^2+y^2+z^2)$.

2. Sketch the largest region where $f$ is continuous. a) $f(x,y) = \sqrt{x^2+y^2-4}$, b) $f(x,y) = \sin^{-1}(y-x)$.

3. a) Find an equation for the level curve of the function $f$ that passes through the point $P$. i) $f(x,y) = \int_x^y \frac{dt}{t^2+1}$, $P(-\sqrt{3},\sqrt{3})$. ii) $f(x,y) = \sum_{n=0}^{\infty}(x/y)^n$, $P(1,2)$. b) Find an equation for the level surface of the function $f$ that passes through the point $P$. i) $f(x,y,z) = \sum_{n=1}^{\infty}\frac{(-1)^n(xyz)^n}{n}$, $P(\sqrt{2},1,1/\sqrt{2})$. ii) $f(x,y,z) = \int_x^y \frac{d\theta}{\sqrt{1-\theta^2}} + \int_{\sqrt{2}}^z \frac{dt}{t\sqrt{t^2-1}}$, $P(0,1/2,2)$. c) Identify the level surfaces of $f(x,y,z) = \ln(x^2+y^2+z^2)$ for $k=-1,\ 0,\ 1$.

4. a) Let $f(x,y,z) = x^2y^3\sin(x^3z^2)$. i) Find $f(y,z,x)$ and $f(z,x,y)$. ii) Find $f_x(x,y,z)$, $f_y(x,y,z)$ and $f_z(x,y,z)$.
b) Use implicit partial differentiation to find $\partial x/\partial y$ and $\partial x/\partial z$ if $xz+y\ln x - x^2+4=0$ defines $x$ as a function of $y$ and $z$. c) If $x = v\ln u$, $y = u\ln v$, use implicit partial differentiation to find $u_x$, $v_x$, $u_y$ and $v_y$. If we set $z=\tan(2u-3v)$, use the chain rule to find $z_x$ and $z_y$. d) answer the same questions as in c) if $x=u^2-v^2$, $y=u^2-v$, and $z=u^2+v^2$.

5. Evaluate each limit.

$$\text{a)} \lim_{(x,y,z)\to(-1,2,1)} \frac{xz^2}{\sqrt{x^2+2y^2+3z^2}},\ \text{b)} \lim_{(x,y)\to(1,1)} \frac{x^2-2xy+y^2}{x^2-y^2},\ \text{c)} \lim_{(x,y)\to(-1,1)} \frac{2x^3+3x^2y-2xy^2-3y^3}{2x^2+xy-y^2},$$

$$\text{d)} \lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2},\ \text{e)} \lim_{(x,y,z)\to(2,2,1)} \frac{\sin(2x-5y+6z)}{(2x-5y+6z)(y+z)},\ \text{f)} \lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{(x^2+y^2)^2}.$$

6. Find an equation for the tangent plane and parametric equations for the normal line to the given surface at the given point. a) $2x^2-2y^2+4z^2=4$, $P(-1,1,1)$. b) $\dfrac{x+2y}{2y+z}=1$, $P(1,\sqrt{7},-2)$.

7. i) Find parametric equations for the tangent line to the curve of intersection of the given surfaces at the given point. a) $z=x^2+y^2$ and $z=8-x^2-y^2$, $P(\sqrt{2},-\sqrt{2},4)$. b) $x^2+y^2=2$ and $x+2y+3z=6$, $P(1,1,1)$. ii) Find all points on the ellipsoid $2x^2+3y^2+4z^2=9$ at which the tangent plane to the ellipsoid is parallel to the plane $x-2y+3z=5$.

8. Let $f(x,y) = \begin{cases} \dfrac{x^2y}{x^2+y^2} - 2x+3y, & (x,y)\neq(0,0), \\ 0, & (x,y)=(0,0). \end{cases}$

a) Find $f_x(0,0)$, and $f_y(0,0)$. b) Show that $f$ is continuous at (0,0). c) Is $f$ differentiable at $(0,0)$?

9. a) Write down the definition of " $f$ is differentiable at $(x_0,y_0)$". b) Use the definition in a) to show that the function $f$ given by $f(x,y)=2x-3xy$ is differentiable at the point $(1,-2)$.

10. a) Let $f(x,y,z)=x^3e^{yz}$. i) Find the differential $df$. ii) Find the local linear approximation for $f$ about $P(1,-1,-1)$, and use it to approximate $f(Q)$ with $Q(0.99,-1.01,-0.98)$. b) Answer the same questions for i) $f(x,y,z)=yz\ln(xy)$, $P(e,1,1)$ and $Q(2.72,0.99,1.01)$. ii) $f(x,y,z)=\tan^{-1}(xyz)$, $P(1,1,1)$ and $Q(0.98,1.01,0.99)$

11. Let $g(x,y,z)=xye^{x+yz}$. a) Find the gradient of $g$ at $P(1,1,-1)$. b) Find a unit vector in the direction in which $g$ decreases most rapidly at the point $Q(-1,1,1)$, and find the rate of change of $g$ at $Q$ in that direction. c) Find the directional derivative of $g$ in the direction of the vector $\overrightarrow{a}=2\overrightarrow{i}-3\overrightarrow{j}+4\overrightarrow{k}$ at the point $A(1,-1,1)$.

12. Find all the first partial derivatives of $f$ if a) $f(x,y)=\log_x(y)$, b) $f(x,y)=\int_x^y g(t)\,dt$, c) $f(x,y,z)=yz\ln(xy)$, d) If $z$ is defined implicitly by the relation $f(x,y,z)=0$ as a differentiable function of $x$ and $y$, where $f$ is a differentiable function with $f_z(x,y,z)\neq 0$ for all allowable points (x,y,z), use implicit differentiation and the chain rule to find the partial derivatives $z_x$ and $z_y$.

13. Review the true/false problems in the text.

14. a) Find an equation for, and identify the surface that results when the elliptic cone $4x^2+9y^2-25z^2=0$ is reflected about the plane: i) $x=0$, ii) $y=0$, iii) $z=0$, iv) $x=y$, v) $y=z$, vi) $z=x$.

b) Identify each quadric surface: i) $9x^2-4y^2-z^2=1$, ii) $y=x^2-z^2$, iii) $z=(x-2)^2+4(y+3)^2$, iv) $9x^2+y^2+4z^2-18x+2y+16z=10$, v) $z^2=4x^2+y^2+8x-2y+4z$, vi) $4x^2-y^2+16(z-2)^2=100$.

15. Given that the directional derivative of a function $f$ at the point $P(3,-2,1)$ in the direction of the vector $\overrightarrow{a} = 2\overrightarrow{i}-\overrightarrow{j}-2\overrightarrow{k}$ is $-5$ and that $||\nabla f(P)||=5$, find $\nabla f(P)$.