MAC 2313 (Calculus III)
Test 2 Review- Spring 2015

1. Describe the domain of the function f in words. a) $f(x,y,z)=\ln(z^2-x^2-y^2)$, b) $f(x,y,z)=\cos^{-1}(x^2+y^2+z^2)$.

2. Sketch the largest region where f is continuous. a) $f(x,y)=\sqrt{x^2+y^2-4}$, b) $f(x,y)=\sin^{-1}(y-x)$.

3. a) Find an equation for the level curve of the function f that passes through the point P. i) $f(x,y)=\int_x^y \frac{dt}{t^2+1}$, $P(-\sqrt{3},\sqrt{3})$. ii) $f(x,y)=\displaystyle\sum_{n=0}^{\infty}(x/y)^n$, $P(1,2)$. b) Find an equation for the level surface of the function f that passes through the point P. i) $f(x,y,z)=\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(xyz)^n}{n}$, $P(\sqrt{2},1,1/\sqrt{2})$. ii) $f(x,y,z)=\int_x^y \frac{d\theta}{\sqrt{1-\theta^2}}+\int_{\sqrt{2}}^z \frac{dt}{t\sqrt{t^2-1}}$, $P(0,1/2,2)$.
c) Identify the level surfaces of $f(x,y,z)=\ln(x^2+y^2+z^2)$ for $k=-1,\ 0,\ 1$.

4. a) Let $f(x,y,z)=x^2y^3\sin(x^3z^2)$. i) Find $f(y,z,x)$ and $f(z,x,y)$. ii) Find $f_x(x,y,z)$, $f_y(x,y,z)$ and $f_z(x,y,z)$.
b) Use implicit partial differentiation to find $\partial x/\partial y$ and $\partial x/\partial z$ if $xz+y\ln x-x^2+4=0$ defines x as a function of y and z. c) If $x=v\ln u$, $y=u\ln v$, use implicit partial differentiation to find u_x, v_x, u_y and v_y. If we set $z=\tan(2u-3v)$, use the chain rule to find z_x and z_y. d) answer the same questions as in c) if $x=u^2-v^2$, $y=u^2-v$, and $z=u^2+v^2$.

5. Evaluate each limit.

$$\text{a)}\lim_{(x,y,z)\to(-1,2,1)}\frac{xz^2}{\sqrt{x^2+2y^2+3z^2}},\ \text{b)}\lim_{(x,y)\to(1,1)}\frac{x^2-2xy+y^2}{x^2-y^2},\ \text{c)}\lim_{(x,y)\to(-1,1)}\frac{2x^3+3x^2y-2xy^2-3y^2}{2x^2+xy-y^2},$$

$$\text{d)}\lim_{(x,y,z)\to(0,0,0)}\frac{xyz}{x^2+y^2+z^2},\ \text{e)}\lim_{(x,y,z)\to(2,2,1)}\frac{\sin(2x-5y+6z)}{(2x-5y+6z)(y+z)},\ \text{f)}\lim_{(x,y)\to(0,0)}\frac{1-\cos(x^2+y^2)}{(x^2+y^2)^2}.$$

6. Find an equation for the tangent plane and parametric equations for the normal line to the given surface at the given point. a) $3x^2-2y^2+4z^2=5$, $P(-1,1,1)$. b) $\dfrac{x+2y}{2y+z}=1$, $P(1,2,1)$.

7. Find parametric equations for the tangent line to the curve of intersection of the given surfaces at the given point. a) $z=x^2+y^2$ and $z=8-x^2-y^2$, $P(\sqrt{2},-\sqrt{2},4)$. b) $x^2+y^2=2$ and $x+2y+3z=6$, $P(1,1,1)$.

8. Let $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}-2x+3y, & (x,y)\neq(0,0),\\ 0, & (x,y)=(0,0).\end{cases}$
a) Find $f_x(0,0)$, and $f_y(0,0)$. b) Show that f is not continuous at (0,0). c) Is f differentiable at $(0,0)$?

9. a) Write down the definition of " f is differentiable at (x_0,y_0)". b) Use the definition in a) to show that the function f given by $f(x,y)=2x-3xy$ is differentiable at the point $(1,-2)$.

10. a) Let $f(x,y,z)=x^3e^{yz}$. i) Find the differential df. ii) Find the local linear approximation for f about $P(1,-1,-1)$, and use it to approximate $f(Q)$ with $Q(0.99,-1.01,-0.98)$. b) Answer the same questions for i) $f(x,y,z)=yz\ln(xy)$, $P(e,1,1)$ and $Q(2.72,0.99,1.01)$. ii) $f(x,y,z)=\tan^{-1}(xyz)$, $P(1,1,1)$ and $Q(0.98,1.01,0.99)$

11. Let $g(x,y,z)=xye^{x+yz}$. a) Find the gradient of g at $P(1,1,-1)$. b) Find a unit vector in the direction in which g decreases most rapidly at the point $Q(-1,1,1)$, and find the rate of change of g at Q in that direction. c) Find the directional derivative of g in the direction of the vector $\overrightarrow{a}=2\overrightarrow{i}-3\overrightarrow{j}+4\overrightarrow{k}$ at the point $A(1,-1,1)$.

12. Find all the critical points of f and classify them as points of local minimum, local maximum, or saddle points. a) $f(x,y)=xy+2x-\ln(x^2y)$, b) $f(x,y)=x^3-y^3-2xy+6$, c) $f(x,y)=4xy+x^4+y^4$, d) $f(x,y)=x^3y^3$, e) $f(x,y)=2y^2x-x^2y+4xy$.

13. Find all the first partial derivatives of f if a) $f(x,y)=\log_x(y)$, b) $f(x,y)=\int_x^y g(t)\,dt$, c) $f(x,y,z)=yz\ln(xy)$, d) $f(x,y,z)=xy\sec(xz)$.

14. a) Find the point on the paraboloid $z=x^2+y^2+10$ that is closest to the plane $x+2y-z=0$. b) Find three positive numbers whose sum is 48 and their product is as large as possible. c) Probems 42 and 44 in 13.8, p. 987.

15. Review the true/false problems in the text.