MAC 2311 (Calculus I) — Answers
Test 3, Wednesday October 28, 2015

Name: **PID:**

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you must show all your work to deserve the full mark assigned to any question. Guessing the correct answers won't earn you any credits. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University administration. Total=60 points. Good Luck!

1. [8] a)Find the local linear approximation for the function f defined by $f(x) = \cos x$ at $x = \pi/3$. b) Use it to approximate $\cos(59°)$.

a) $L(x) = f(\frac{\pi}{3}) + f'(\frac{\pi}{3})(x - \frac{\pi}{3})$; $L(x) = \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \frac{\pi}{3})$

$f'(x) = -\sin x$

b) $\cos(59°) = \cos(\frac{59\pi}{180}) \approx \frac{1}{2} - \frac{\sqrt{3}}{2}(\frac{59\pi}{180} - \frac{60\pi}{180}) = \frac{1}{2} - \frac{\sqrt{3}}{2}(-\frac{\pi}{180})$

$\approx \frac{1}{2} + \frac{\sqrt{3}\pi}{360}$

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2. [10] Evaluate each limit.

a) $\lim_{x\to+\infty} \frac{\ln(2x)}{\ln(3x)} = \frac{+\infty}{+\infty}$

$\overset{H.R}{=} \lim_{x\to+\infty} \frac{2/2x}{3/3x}$

$= \lim_{x\to+\infty} \frac{1/x}{1/x}$

$= \lim_{x\to+\infty} \frac{x}{x}$

$= 1$

b) $\lim_{x\to 0} \frac{e^x - 1 - x}{1 - \cos(2x)} = \frac{0}{0}$

$\overset{H.R}{=} \lim_{x\to 0} \frac{e^x - 1}{2\sin(2x)} = \frac{0}{0}$

$\overset{H.R}{=} \lim_{x\to 0} \frac{e^x}{4\cos(2x)}$

$= \frac{1}{4}$

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3. [8] Three quantities x, y and z all depend on time, and are related by the equation $z^3 - 3xy = x^2$. If x is increasing at the constant rate of 2 in/s and y is decreasing at the constant rate of 3 in/s, how fast is z changing when $x = 3$ in and $y = 2$ in? Is z increasing or decreasing?

$\frac{d}{dt}(z^3 - 3xy) = \frac{d}{dt}(x^2)$; $3z^2\frac{dz}{dt} - 3x\frac{dy}{dt} - 3y\frac{dx}{dt} = 2x\frac{dx}{dt}$

$x = 3,\ y = 2 \to z^3 - 18 = 9 \to z^3 = 27 \to z = 3$; $\frac{dx}{dt} = 2$ in/s, $\frac{dy}{dt} = -3$ in/s

$27\frac{dz}{dt} - 9(-3) - 6(2) = 6(2) \to 27\frac{dz}{dt} = 24 - 27 = -3$

so $\frac{dz}{dt} = -\frac{1}{9}$ in/s. z is decreasing at the rate of $\frac{1}{9}$ in/s

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4. [10, **Bonus**] Let $f(x) = \cos^2 x - \sin x$. Find the absolute maximum and minimum values of f on the interval $[0, 2\pi]$, and state where they occur.

$f'(x) = -2\sin x\cos x - \cos x$

$= -\cos x\,(2\sin x + 1)$

$f'(x) = 0 \to \cos x = 0 \to x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$ both lie in $[0, 2\pi]$

or $2\sin x + 1 = 0 \to \sin x = -\frac{1}{2} \to x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

Now $f(\frac{\pi}{2}) = -1$, $f(\frac{3\pi}{2}) = 1$, $f(\frac{7\pi}{6}) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4} = f(\frac{11\pi}{6})$

$f(0) = 1 = f(2\pi)$.

Absolute minimum $= -1$ at $x = \frac{\pi}{2}$

Absolute maximum $= \frac{5}{4}$ at $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$.

5. [8] Decide whether the statement is true or false. No explanation needed.

a) If $f'(\pi) = 0$, then f has a relative maximum or a relative minimum at $x = \pi$. False; pick $f(x) = (x-\pi)^3$

b) If $f''(-28) = 0$, then the point $(-28, f(-28))$ is an inflection point of f. False; pick $f(x) = (x+28)^4$

c) If f has a relative minimum at $x = 4$, then $f(4) \le f(4.1)$. False

d) If f' is decreasing on [3,7] and f' is decreasing on [7,10], then $(7, f(7))$ is an inflection point of f. False by definition of concavity

e) If $f'(3\pi) = 0$ and $f''(3\pi) > 0$, then f has a relative minimum at $x = 3\pi$. True, by Second Derivative Test

f) If $f'(5)$ does not exist, then $x = 5$ is a critical point of f. , by definition of critical point

g) If $f'(-8) < 0$, then f is decreasing on the interval [-9,-7]. False, pick $f(x) = (x + \frac{15}{2})^2$

h) If f has a relative minimum at $x = -\sqrt{2}$, then $x = -\sqrt{2}$ is a critical point of f. True, by Theorem 4.2.2 in text

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6. [16] a) Find all the intercepts, asymptotes, intervals of increase, decrease, concavity, inflection points of the function f defined by $f(x) = x^3 - 3x - 2$. b) Find and classify all the critical points of f as points of local maximum, local minimum or neither. c) Sketch the graph of f.

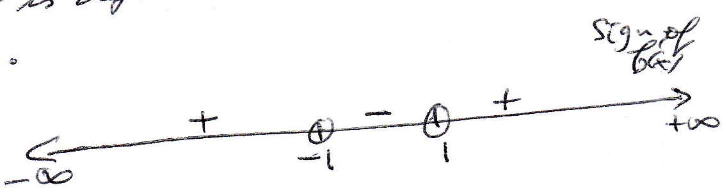
a) x-intercepts: $x^3 - 3x - 2 = 0$; $x = -1$ is a solution, so

$x^3 - 3x - 2 = (x+1)(x^2 - x - 2) = (x+1)(x+1)(x-2)$

x-intercepts: $x = -1$, $x = 2$

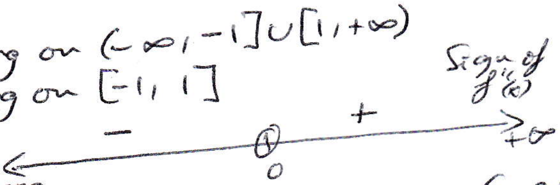
y-intercept $= f(0) = -2$

Asymptotes: there are none since f is defined everywhere, and $\lim_{x\to-\infty} f(x) = -\infty$, $\lim_{x\to+\infty} f(x) = +\infty$.



$f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$

- f is increasing on $(-\infty, -1] \cup [1, +\infty)$
- f is decreasing on $[-1, 1]$



$f''(x) = 6x$

- f is CU on $(0, +\infty)$ and f is CD on $(-\infty, 0)$; IP: $(0, f(0)) = (0, -2)$

b) C.P.s: $x = 1$, $x = -1$;
- f has a local maximum at $x = -1$, by FDT
- f has a local minimum at $x = 1$, by FDT

c)