MAC 2311 (Calculus I) — Key

Test 3 , Wednesday March 25, 2015

Name: PID:

Remember that no documents or calculators are allowed during the exam. Be as precise as possible in your work; you must show all your work to deserve the full mark assigned to any question. Guessing the correct answers won't earn you any credits. Do not cheat, otherwise I will be forced to give you a zero and report your act of cheating to the University administration. Total=60 points. Good Luck!

1. [8] a)Find the local linear approximation for the function f defined by $f(x) = \sqrt[3]{x}$ about $x_0 = 8$. b) Use it to approximate $\sqrt[3]{7.88}$.

a) $L(x) = f(8) + (x-8)f'(8)$. $f'(x) = \frac{1}{3x^{\frac{2}{3}}}$ b) $f(8) = 2$, $f'(8) = \frac{1}{3(\sqrt[3]{8})^2} = \frac{1}{12}$

$= 2 + \frac{(x-8)}{12}$

$\sqrt[3]{7.88} \approx 2 + \frac{7.88-8}{12} = 2 - \frac{0.12}{12} = 2 - 0.01 = 1.99$

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2. [10] Evaluate each limit.

a) $\lim_{x\to 0}(1-2x)^{\frac{1}{x}} = e^{\lim_{x\to 0} \frac{\ln(1-2x)}{x}} \overset{H.R}{=} e^{\lim_{x\to 0} \frac{-2/(1-2x)}{1}}$

$= e^{-2}$

b) $\lim_{x\to -\infty} x\sin(1/2x) = \lim_{x\to-\infty} \frac{\sin(1/2x)}{2\left(\frac{1}{2x}\right)}$

$= \frac{1}{2}\lim_{u\to 0^-} \frac{\sin u}{u}$, $u = \frac{1}{2x}$

$= \frac{1}{2}(1)$

$= \frac{1}{2}$

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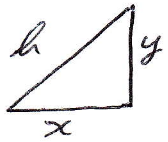
3. [8] A right triangle has a hypotenuse of length h, while the lengths of the two other sides are x and y respectively. If h is increasing at a rate of 2 in/s and x is decreasing at a rate of 1.5 in/s, how fast is y changing when $h = 10$ in and $x = 6$ in?



$h^2 = x^2 + y^2$; $\frac{d}{dt}(h^2) = \frac{d}{dt}(x^2+y^2)$; $2h\frac{dh}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$

$\frac{dh}{dt} = 2$ in/s $\quad \frac{dx}{dt} = -1.5$/s

$h = 10$, and $x = 6 \longrightarrow y^2 = 10^2 - 6^2 = 64 \to y = 8$

$y\frac{dy}{dt} = +h\frac{dh}{dt} - x\frac{dx}{dt}$; so $8\frac{dy}{dt} = 10(2) - 6(-1.5) = 29$; So $\frac{dy}{dt} = \frac{29}{8}$ in/s

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4. [10, **Bonus**] a) If $y = x\sqrt{3+x}$, find Δy and dy at $x = 1$. b) Assuming that $\Delta x = dx$, use the differential dy to approximate Δy when x changes from $x = 0.98$ to $x = 1$.

a) $\Delta y = (1+\Delta x)\sqrt{3+1+\Delta x} - 1\sqrt{3+1} = (1+\Delta x)\sqrt{4+\Delta x} - 2$

$dy(x) = y'(x)dx = \left(\sqrt{3+x} + \frac{x}{2\sqrt{3+x}}\right)dx$

$dy = \left(2 + \frac{1}{4}\right)dx = \frac{9}{4}dx$ at $x=1$

b) $\Delta y \approx dy = \frac{9}{4}dx = \frac{9}{4}\Delta x$

$\Delta y \approx \frac{9}{4}(0.98-1) = \frac{9}{4}(-0.02) = -\frac{0.09}{2}$

$\Delta y \approx -0.045$

5. [10] Decide whether the statement is true or false. No explanation needed.

a) If f' is decreasing on [-1,2] and f' is increasing on [2,5], then $(2, f(2))$ is an inflection point of f. T, by Definition of IP

b) If $f'(1) = 0$ and $f''(1) > 0$, then f has a relative minimum at $x = 1$. T, by SDT

c) If $f'(6) < 0$, then f is decreasing on the interval [5,7]. False, check with $f(x) = (x - \frac{13}{2})^2$

d) If $f'(-2)$ does not exist, then $x = -2$ is a critical point of f. False, in addition, f must be continuous at $x=-2$

e) If f is increasing on [-4,-1], then $f(-3) < f(-1)$. T, by definition

f) If f has a relative minimum at $x = 7$, then $x = 7$ is a critical point of f. T, by Theorem 4.2.2 in Text

g) If $f'(6) = 0$, then f has a relative maximum or a relative minimum at $x = 6$. False, check with $f(x) = (x-6)^3$

h) If $f''(24) = 0$, then the point $(4, f(24))$ is an inflection point of f. False

i) If f has a relative minimum at $x = -5$, then $f(-5) \leq f(-5.5)$. False

j) If $\lim_{x\to+\infty}(f(x) - r(x)) = 0$, then the curve $y = r(x)$ is an asymptote of f. T, as given in class.

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6. [14] a) Find all the intercepts, asymptotes, intervals of increase, decrease, concavity, inflection points of the function f defined by $f(x) = x^4 - 12x^2$. b) Find and classify all the critical points of f.

a) x-intercepts: solve $x^4 - 12x^2 = 0$ or $x^2(x^2-12) = 0$; so $x = 0$ or $x = \pm\sqrt{12} = \pm 2\sqrt{3}$

y-intercept $= f(0) = 0$

Asymptotes: None

$f'(x) = 4x^3 - 24x = 4x(x^2-6) = 4x(x-\sqrt{6})(x+\sqrt{6})$

Sign of $f'(x)$

$-\infty$ $-$ $(-)(-)(-)$ $-\sqrt{6}$ $+$ $(-)(-)(+)$ 0 $-$ $(+)(-)(+)$ $\sqrt{6}$ $+$ $(+)(+)(+)$ $+\infty$

- f is increasing on $[-\sqrt{6}, 0] \cup [\sqrt{6}, +\infty)$
- f is decreasing on $(-\infty, -\sqrt{6}] \cup [0, \sqrt{6}]$

$f''(x) = 12x^2 - 24 = 12(x^2-2) = 12(x-\sqrt{2})(x+\sqrt{2})$

Sign of $f''(x)$

$-\infty$ $+$ $(-)(-)$ $-\sqrt{2}$ $-$ $(-)(+)$ $\sqrt{2}$ $+$ $(+)(+)$ $+\infty$

- f is CU on $(-\infty, -\sqrt{2}) \cup (\sqrt{2}, +\infty)$
- f is CD on $(-\sqrt{2}, \sqrt{2})$

IP: $(-\sqrt{2}, f(-\sqrt{2}))$ and $(\sqrt{2}, f(\sqrt{2}))$

b) C.Ps: $x = -\sqrt{6}$; 0; $\sqrt{6}$

- f has a local minimum at both $x = -\sqrt{6}$ and $x = \sqrt{6}$, by the FDT.
- f has a local maximum at $x = 0$, by the FDT.