MAC 2313 (Calculus III)
Test 3 Review. The test covers chp. 14, and 15.1 to 15.5.

1. Evaluate each integral.

a) $\int\int_R e^s \ln t \, dA$; R = region in the first quadrant of the st-plane that lies above the curve $s = \ln t$ from $t = 1$ to $t = 2$. b) $\int_0^\pi \int_x^\pi \frac{\sin y}{y} \, dydx$. c) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} \, dydx$. d) $\int_0^{\ln 2} \int_{e^y}^2 e^{x+y} \, dxdy$. e) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \ln(x^2 + y^2 + 1) \, dydx$.

2. Find the volume of the given solid G.

a) G = solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3$. b) G = solid bounded above by the cylinder $x^2 + z^2 = 4$, below by the xy-plane and laterally by the cylinder $x^2 + y^2 = 4$. c) G = solid below the cone $z = \sqrt{x^2 + y^2}$, inside the cylinder $x^2 + y^2 = 2y$, and above $z = 0$. d) G = solid inside the sphere $r^2 + z^2 = 4$ and outside the cylinder $r = 2\cos\theta$.

3. Evaluate each triple integral a) $\int_1^3 \int_x^{x^2} \int_0^{\ln z} xe^y \, dydzdx$. b) $\int_1^2 \int_z^2 \int_0^{y\sqrt{3}} \frac{y}{x^2+y^2} \, dxdydz$. c) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{a \sec\phi} \rho^2 \sin\phi \, d\rho d\phi d\theta$. d) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} e^{-(x^2+y^2+z^2)^{\frac{3}{2}}} \, dzdydx$. e) $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2 \, dzdxdy$.

4. Write down an equivalent integral using the order of integration provided, but do not evaluate.

a) $\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin(2z)}{4-z} \, dydzdx$; xyz and xzy. b) $\int_0^4 \int_0^{4-y} \int_0^{\sqrt{z}} f(x, y, z) \, dxdzdy$; zyx and yxz.

5. Use spherical coordinates to find the volume of the solid G.

a) G = solid within the cone $\phi = \pi/4$ and between the spheres $\rho = 1$ and $\rho = 2$. b) G = solid within the sphere $x^2 + y^2 + z^2 = 9$, outside the cone $z = \sqrt{x^2 + y^2}$. c) G = solid enclosed by the sphere $x^2 + y^2 + z^2 = 8$, and the planes $z = 0$ and $z = \sqrt{2}$. d) G = solid bounded above by the cone $z = 4 - \sqrt{x^2 + y^2}$ and below by the cone $z = \sqrt{x^2 + y^2}$. e) G = solid enclosed by the cylinder $x^2 + y^2 = 3$ and the planes $z = 1$ and $z = 3$.

6. Use cylindrical coordinates to find the volume of the solid: a) that is inside the sphere $r^2 + z^2 = 20$, but not above the paraboloid $z = r^2$. b) bounded above by the paraboloid $z = 8 - x^2 - y^2$ and below by the cone $z = \sqrt{x^2 + y^2}$. c) inside the cylinder $x^2 + y^2 = 4$, below the cone $z = 6 - \sqrt{x^2 + y^2}$ and above the xy-plane.

7. Find the Jacobian $\partial(x, y)/\partial(u, v)$. a) $u = x^2 + y^2$, $v = xy$. b) $u = x^2 - y^2$, $v = 2x - y$.

8. Find the Jacobian $\partial(x, y, z)/\partial(u, v, w)$. a) $u = xy$, $v = yz$, $w = x + z$. b) $x = u - uv$, $y = uv - uvw$, $z = uvw$.

9. Evaluate the integral by making an appropriate change of variables.

a) $\int\int_R \frac{\sin(x-y)}{\cos(x+y)} \, dA$, where R is the triangular region enclosed by the lines $y = 0$, $y = x$, $x + y = \pi/4$.
b) $\int\int_R e^{\frac{(y-x)}{(y+x)}} \, dA$, where R is the region in the first quadrant enclosed by the trapezoid with vertices (0,1), (1,0), (0,4), (4,0).

10. Use the transformation $u = xy$, $v = x^2 - y^2$ to evaluate $\int\int_R (x^4 - y^4)e^{xy} \, dA$, where R is the region in the first quadrant enclosed by the hyperbolas $xy = 1$, $xy = 3$, $x^2 - y^2 = 3$, $x^2 - y^2 = 4$.

11. Let G be the solid defined by the inequalities: $1 - e^x \le y \le 3 - e^x$, $\quad 1 - y \le 2z \le 2 - y$, $\quad y \le e^x \le y + 4$.

a) Using the change of variables $u = e^x + y$, $v = y + 2z$, $w = e^x - y$, find the Jacobian $\partial(x, y, z)/\partial(u, v, w)$ and express it in terms of u, v, and w. b) Find the volume of G using the change of variables in part a). c) Write down the coordinates of the centroid of G, include for each coordinate the appropriate limits of integration, bu do not evaluate any of the triple integrals involved.

12. a) Let G be the solid defined by the inequalities: $\sqrt{x^2 + y^2} \le z \le 20 - x^2 - y^2$. Find the coordinates of the centroid of G. b) Find the mass and center of gravity of the solid G enclosed by the portion of the sphere $x^2 + y^2 + z^2 = 2$ on or above the plane $z = 1$ if the density is $\delta = \sqrt{x^2 + y^2 + z^2}$.

13. a) State the fundamental theorem of line integral. b) Let $F(x, y) = (2xy + x)\vec{i} + (x^2 + 2y)\vec{j}$. b1) Show that F is conservative. b2) Find a potential function φ for F. b3) Evaluate the line integral $\int_{\mathcal{C}} (2xy + x) \, dx + (x^2 + 2y) \, dy$ along the curve $\mathcal{C}$ parametrized by $\vec{r}(t) = \sqrt{1 + t}\,\vec{i} + \sin^{-1} t\,\vec{j}$, $\quad 0 \le t \le 1$.

14. a) Set up, but do not evaluate, two iterated integrals equal to the surface integral $\int\int_\sigma y^2 z dS$, where σ is the portion of the cylinder $x^2 + z^2 = 4$ in the first octant between the planes $y = 0$, $y = 6$, $x = z$, and $x = 2z$. b) Consider the parametric surface given by $\mathbf{r}(u, v) = u\,\vec{i} + u\cos v\,\vec{j} + u \sin v\,\vec{k}$ with $0 \le u \le 4$ and $0 \le v \le \pi$. i) Find the area S of σ. ii) Find the mass M of σ if its density is $\delta(x, y, z) = x^2 + y^2 + z^2$. iii) Evaluate the surface integral $\int\int_\sigma x\sqrt{z} dS$ where

σ is the portion of the paraboloid $z = x^2 + y^2$ in the first octant between the planes $z = 0$ and $z = 4$. iv) a) Find an equation for the tangent plane to the parametric surface σ given by: $\overrightarrow{r}(u, v) = 4u \cos v \overrightarrow{i} + u^2 \overrightarrow{j} + 3u \sin v \overrightarrow{k}$, at the point P corresponding to $(u, v) = (1, \pi/2)$..

15. Let $F(x, y) = (x^3 y + 4e^{-2x}) \overrightarrow{i} + (\frac{x^4}{4} + y^2) \overrightarrow{j}$. a) Show that F is conservative. b) Find a potential function φ for F. c) Evaluate the line integral $\displaystyle\int_{\mathcal{C}} (x^3 y + 4e^{-2x})\, dx + (\frac{x^4}{4} + y^2)\, dy$ along the curve $\mathcal{C}$ parametrized by $\overrightarrow{r}(t) = \cos^3 t \overrightarrow{i} + \sin^3 t \overrightarrow{j}$, $0 \leq t \leq \pi$.

16. a) Let $\mathbf{F}(x, y, z) = (x^2 - 2yx) \overrightarrow{i} + (3y^2 - 2yz) \overrightarrow{j} + (5z^2 - 2xz) \overrightarrow{k}$. Find div$\mathbf{F}$ and curl$\mathbf{F}$. Evaluate the line integral $\int_{\mathcal{C}} \text{curl}\mathbf{F} \cdot \mathbf{dr}$, where $\mathcal{C}$ is the triangle with vertices (0,0,2), (0,2,0) and (2,0,0).

17. Let $\mathcal{C}$ be the curve given by $x = t$, $y = 3t^2$, $z = 6t^3$, $\quad 0 \leq t \leq 1$, and evaluate $\int_{\mathcal{C}} xyz^2\, ds$. b) Evaluate the line integral along $\mathcal{C}$ given by $\mathcal{C} : x = t$, $y = t^2$, $z = 3t^2$, $0 \leq t \leq 1$, $\displaystyle\int_{\mathcal{C}} \sqrt{1 + 30x^2 + 10y}\, ds$. c) Evaluate $\int_{\mathcal{C}} y dx + z dy - x dz$ along the helix $x = \cos(\pi t)$, $\quad y = \sin(\pi t)$, $\quad z = t$ from the point (1,0,0) to (-1,0,1). d) Find the mass of a thin wire shaped in the form of the curve $x = e^t \cos t$, $y = e^t \sin t$, $(0 \leq t \leq 1)$ if the density function δ is proportional to the distance to the origin.

18. a) Find parametric equations for the paraboloid $z = x^2 + y^2$ in terms of the parameters θ and ϕ, where (ρ, θ, ϕ) are spherical coordinates of a point on the surface. b) Find a parametric representation of the portion of the sphere $x^2 + y^2 + z^2 = 9$ on or above the plane $z = 2$ in terms of the parameters r and θ, where (r, θ, z) are the cylindrical coordinates of a point on the surface.

19. a) Use Green's theorem to evaluate the line integral $\int_{\mathcal{C}} (4y + \cos(1 + e^{\sin x}))\, dx + (2x - \sec^2 y)\, dy$, where $\mathcal{C}$ is the circle $x^2 + y^2 = 9$ going from (0,3) to (0,3) counterclockwise.

20. Review the Fundamental Theorem of Line Integral and Green's Theorem.