MAC 2313 (Calculus III)
Test 3 Review

Test 3 will cover sections 13.9,14.1 to 14.3, 14.5 and 14.6.

1. Find the point B on the plane $x+2y+3z=12$ that is closest to the point $A(1,2,-3)$. Find the distance between A and B.

2. A space probe in the shape of the ellipsoid $4x^2+y^2+4z^2=16$ enters the Earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point $(x,y,z$ on the probe's surface is $T(x,y,z)=8x^2+4yz-16z+600$. Find the hottest point on the probe's surface.

3. a) Find the point on the sphere $x^2+y^2+z^2=4$ farthest from the point $D(1,-1,1)$. b) Find the minimum distance from the surface $x^2+y^2-z^2=1$ to the origin.

4. Evaluate each integral.

a) $\int\int_R e^s \ln t\, dA$; $R=$ region in the first quadrant of the st-plane that lies above the curve $s=\ln t$ from $t=1$ to $t=2$. b) $\int_0^\pi\int_x^\pi \frac{\sin y}{y}\,dydx$. c) $\int_0^2\int_0^{4-x^2}\frac{xe^{2y}}{4-y}\,dydx$. d) $\int_0^{\ln 2}\int_{e^y}^2 e^{x+y}\,dxdy$. e) $\int_{-1}^1\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\ln(x^2+y^2+1)\,dydx$.

5. Find the volume of the given solid G.

a) $G=$ solid in the first octant bounded by the coordinate planes, the cylinder $x^2+y^2=4$ and the plane $y+z=3$. b) $G=$ solid bounded above by the cylinder $x^2+z^2=4$, below by the xy-plane and laterally by the cylinder $x^2+y^2=4$. c) $G=$ solid below the cone $z=\sqrt{x^2+y^2}$, inside the cylinder $x^2+y^2=2y$, and above $z=0$. d) $G=$ solid inside the sphere $r^2+z^2=4$ and outside the cylinder $r=2\cos\theta$.

6. Evaluate each triple integral a) $\int_1^3\int_x^{x^2}\int_0^{\ln z} xe^y\,dydzdx$. b) $\int_1^2\int_z^2\int_0^{y\sqrt{3}}\frac{y}{x^2+y^2}\,dxdydz$. c) $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{a\sec\phi}\rho^2\sin\phi\,d\rho d\phi d\theta$. d) $\int_{-1}^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} e^{-(x^2+y^2+z^2)^{\frac{3}{2}}}\,dzdydx$. e) $\int_0^2\int_0^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2\,dzdxdy$.

7. Write down an equivalent integral using the order of integration provided, but do not evaluate.

a) $\int_0^2\int_0^{4-x^2}\int_0^x\frac{\sin(2z)}{4-z}\,dydzdx$; xyz and xzy. b) $\int_0^4\int_0^{4-y}\int_0^{\sqrt{z}} f(x,y,z)\,dxdzdy$; zyx and yxz.

8. Use spherical coordinates to find the volume of the solid G.

a) $G=$ solid within the cone $\phi=\pi/4$ and between the spheres $\rho=1$ and $\rho=2$. b) $G=$ solid within the sphere $x^2+y^2+z^2=9$, outside the cone $z=\sqrt{x^2+y^2}$. c) $G=$ solid enclosed by the sphere $x^2+y^2+z^2=8$, and the planes $z=0$ and $z=\sqrt{2}$. d) $G=$ solid bounded above by the cone $z=4-\sqrt{x^2+y^2}$ and below by the cone $z=\sqrt{x^2+y^2}$. e) $G=$ solid enclosed by the cylinder $x^2+y^2=3$ and the planes $z=1$ and $z=3$.

9. Use cylindrical coordinates to find the volume of the solid: a) that is inside the sphere $r^2+z^2=20$, but not above the paraboloid $z=r^2$. b) bounded above by the paraboloid $z=8-x^2-y^2$ and below by the cone $z=\sqrt{x^2+y^2}$. c) inside the cylinder $x^2+y^2=4$, below the cone $z=6-\sqrt{x^2+y^2}$ and above the xy-plane. d) inside the surface $r^2+z^2=4$ and outside the surface $r=2\cos\theta$.

10. Evaluate each integral using polar coordinates

a) $\int_0^2\int_0^{\sqrt{2x-x^2}}\sqrt{x^2+y^2}\,dydx$. b) $\int_0^a\int_0^{\sqrt{a^2-y^2}}\frac{dxdy}{(1+x^2+y^2)^{\frac{3}{2}}}$. c) $\int_0^1\int_y^{\sqrt{y}}\sqrt{x^2+y^2}dxdy$.