MAC 2313 (Calculus III)
Test 3 Review

Test 3 will cover sections 13.8,13.9,14.1 to 14.3, 14.5 to 14.8.

1. Find the point B on the plane $x+2y+3z=12$ that is closest to the point $A(1,2,-3)$. Find the distance between A and B.

2. A space probe in the shape of the ellipsoid $4x^2+y^2+4z^2=16$ enters the Earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point $(x,y,z$ on the probe's surface is $T(x,y,z)=8x^2+4yz-16z+600$. Find the hottest point on the probe's surface.

3. a) Find the point on the sphere $x^2+y^2+z^2=4$ farthest from the point $D(1,-1,1)$. b) Find the minimum distance from the surface $x^2+y^2-z^2=1$ to the origin.

4. Evaluate each integral.

a) $\int\int_R e^s \ln t\, dA$; $R=$ region in the first quadrant of the st-plane that lies above the curve $s=\ln t$ from $t=1$ to $t=2$. b) $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\, dydx$. c) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dydx$. d) $\int_0^{\ln 2} \int_{e^y}^2 e^{x+y}\, dxdy$. e) $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \ln(x^2+y^2+1)\, dydx$.

5. Find the volume of the given solid G.

a) $G=$ solid in the first octant bounded by the coordinate planes, the cylinder $x^2+y^2=4$ and the plane $y+z=3$.
b) $G=$ solid bounded above by the cylinder $x^2+z^2=4$, below by the xy-plane and laterally by the cylinder $x^2+y^2=4$. c) $G=$ solid below the cone $z=\sqrt{x^2+y^2}$, inside the cylinder $x^2+y^2=2y$, and above $z=0$.
d) $G=$ solid inside the sphere $r^2+z^2=4$ and outside the cylinder $r=2\cos\theta$.

6. Evaluate each triple integral a) $\int_1^3 \int_x^{x^2} \int_0^{\ln z} xe^y\, dydzdx$. b) $\int_1^2 \int_z^2 \int_0^{y\sqrt{3}} \frac{y}{x^2+y^2}\, dxdydz$. c) $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{a\sec\phi} \rho^2 \sin\phi\, d\rho d\phi d\theta$.
d) $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} e^{-(x^2+y^2+z^2)^{\frac{3}{2}}}\, dzdydx$. e) $\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} z^2\, dzdxdy$.
f) $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{x^2+y^2}} \sqrt{x^2+y^2+z^2}\, dzdydx$

7. Write down an equivalent integral using the order of integration provided, but do not evaluate.

a) $\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin(2z)}{4-z}\, dydzdx$; xyz and xzy. b) $\int_0^4 \int_0^{4-x} \int_0^{\sqrt{y}} f(x,y,z)\, dzdydx$; xzy and yxz.

8. Use spherical coordinates to find the volume of the solid G.

a) $G=$ solid within the cone $\phi=\pi/4$ and between the spheres $\rho=1$ and $\rho=2$. b) $G=$ solid within the sphere $x^2+y^2+z^2=9$, outside the cone $z=\sqrt{x^2+y^2}$. c) $G=$ solid enclosed by the sphere $x^2+y^2+z^2=8$, and the planes $z=0$ and $z=\sqrt{2}$. d) $G=$ solid bounded above by the cone $z=4-\sqrt{x^2+y^2}$ and below by the cone $z=\sqrt{x^2+y^2}$. e) $G=$ solid enclosed by the cylinder $x^2+y^2=3$ and the planes $z=1$ and $z=3$.

9. Use cylindrical coordinates to find the volume of the solid: a) that is inside the sphere $r^2+z^2=20$, but not above the paraboloid $z=r^2$. b) bounded above by the paraboloid $z=8-x^2-y^2$ and below by the cone $z=\sqrt{x^2+y^2}$. c) inside the cylinder $x^2+y^2=4$, below the cone $z=6-\sqrt{x^2+y^2}$ and above the xy-plane. d) inside the surface $r^2+z^2=4$ and outside the surface $r=2\cos\theta$.

10. Evaluate each integral using polar coordinates
a) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$. b) $\int_0^a \int_0^{\sqrt{a^2-y^2}} \frac{dxdy}{(1+x^2+y^2)^{\frac{3}{2}}}$. c) $\int_0^1 \int_y^{\sqrt{y}} \sqrt{x^2+y^2} dxdy$.

11. Find all the critical points of f and classify them as points of local minimum, local maximum, or saddle points. a) $f(x,y)=xy+2x-\ln(x^2y)$, b) $f(x,y)=x^3-y^3-2xy+6$, c) $f(x,y)=4xy+x^4+y^4$, d) $f(x,y)=x^3y^3$, e) $f(x,y)=2y^2x-x^2y+4xy$.

12. Find the point on the paraboloid $z=x^2+y^2+10$ that is closest to the plane $x+2y-z=0$.

13. a) Let G be the solid defined by the inequalities: $\sqrt{x^2+y^2}\le z\le 20-x^2-y^2$. Find the coordinates of the centroid of G. b) Find the mass and center of gravity of the solid G enclosed by the portion of the sphere $x^2+y^2+z^2=2$ on or above the plane $z=1$ if the density is $\delta=\sqrt{x^2+y^2+z^2}$.

14. a) Find the Jacobian $\partial(x,y)/\partial(u,v)$. a) $u=x^2+y^2$, $v=xy$. b) $u=x^2-y^2$, $v=2x-y$. b) Find the Jacobian $\partial(x,y,z)/\partial(u,v,w)$. b1) $u=xy$, $v=yz$, $w=x+z$. b2) $x=u-uv$, $y=uv-uvw$, $z=uvw$.

15. Evaluate the integral by making an appropriate change of variables.

a) $\int\int_R \frac{\sin(x-y)}{\cos(x+y)}\, dA$, where R is the triangular region enclosed by the lines $y=0$, $y=x$, $x+y=\pi/4$.

b) $\int\int_R e^{\frac{(y-x)}{(y+x)}}\, dA$, where R is the region in the first quadrant enclosed by the trapezoid with vertices (0,1), (1,0), (0,4), (4,0).

16. Use the transformation $u = xy$, $v = x^2 - y^2$ to evaluate $\int\int_R (x^4 - y^4)e^{xy}\, dA$, where R is the region in the first quadrant enclosed by the hyperbolas $xy = 1$, $xy = 3$, $x^2 - y^2 = 3$, $x^2 - y^2 = 4$.

17. Let G be the solid defined by the inequalities: $1 - e^x \leq y \leq 3 - e^x$, $\quad 1 - y \leq 2z \leq 2 - y$, $\quad y \leq e^x \leq y + 4$.

a) Using the change of variables $u = e^x + y$, $v = y + 2z$, $w = e^x - y$, find the Jacobian $\partial(x, y, z)/\partial(u, v, w)$ and express it in terms of u, v, and w. b) Find the volume of G using the change of variables in part a). c) Write down the coordinates of the centroid of G, include for each coordinate the appropriate limits of integration, bu do not evaluate any of the triple integrals involved.