MAC 2311 (Calculus I)

Test 4 Review: The test will not include parametric equations; you may disregard problem 8 for Test 4

1. a) A rectangular field with an area of 3200 ft^2 is to be fenced off. Two opposite sides will use fencing costing \$1 per foot and the remaining sides will use fencing costing \$2 per foot. Find the dimensions of the field with least cost. b) The perimeter of a sector of a circle is 12 ft. Find the radius so that the area of the sector is a maximum. Find the angle of the sector of maximum area.

2. A rectangular box with a square base is to have a volume of 2000 cm^3. It costs twice as much per square centimeter for the top and bottom as it does for the sides. Find the dimensions of least cost.

3. The shoreline of Circle Lake is a circle with diameter 2 mi. Nancy's training routine begins at point E on the Eastern shore of the lake. She jogs along the north shore to a point P, and then swims the straight line distance,if any, from P to the point W diametrically opposite E. Nancy swims at a rate of 2 mi/h and jogs a 8 mi/h. How far should Nancy jog in order to complete her training routine in a) the least amount of time? b) the greatest amount of time?

4. The function $s(t) = \frac{t^4}{4} - \frac{7}{3}t^3 + 7t^2 - 8t + 1$, $t \geq 0$ denotes the position of a particle moving along a straight line, where s is in feet and t in seconds. a) find the velocity and acceleration functions. Find the position, velocity, speed, and acceleration at time $t = 3$, c) When is the particle stopped? d) When is the particle speeding up? Slowing down? e) Find the total distance traveled from time $t = 0$ to time $t = 3$. f) Give a schematic picture of the motion.

5. a) State the Mean-value theorem. b) Let $f(x) = x - \tan^{-1} x, \quad 0 \leq x \leq 1$. Show that f satisfies all the requirements of the Mean-value theorem on the given interval, and find all x_0 in $(0, 1)$ that satisfy the conclusion of the theorem. c) Use the MVT to show that $\sqrt{y} - \sqrt{x} < \dfrac{y-x}{2\sqrt{x}}$, for all $0 < x < y$. Derive from the last inequality that for all $0 < x < y$, one has $\sqrt{xy} < \dfrac{x+y}{2}$. d) Use the MVT to prove: j) $\dfrac{x}{x^2+1} < \tan^{-1} x < x$, for all $x > 0$, jj) $\frac{20}{9} < \sqrt{5} < \frac{9}{4}$, jjj) $\frac{143}{64} < \sqrt{5} < \frac{179}{80}$, (Hint. You may use $f(x) = \sqrt{x}$, $a = 4$ and $b = 5$). e) An automobile travels 4 mi along a straight road in 5 min. Show that the speedometer reads exactly 48 mi/h at least once during the trip. f) Let $f(x) = x^{\frac{2}{3}}$, $a = -1$ and $b = 8$. i) Show that there is no x_0 in (a, b) such that $f'(x_0) = \dfrac{f(b) - f(a)}{b - a}$. ii) Explain why the result in i) does not contradict the MVT.

6. Evaluate each indefinite integral using algebraic manipulations or an appropriate substitution.

a) $\displaystyle\int (x^3 - 5x^2 + 7x - 8)\sqrt{x}\, dx$, b) $\displaystyle\int \frac{1}{1 + \cos(2t)}\, dt$, c) $\displaystyle\int \frac{1}{1 + \sin x}\, dx$, d) $\displaystyle\int \frac{1}{x\sqrt{x^2 - 1}}\, dx$

e) $\displaystyle\int \frac{1}{2\sqrt{1 - x^2}} - \frac{(x^4 + 3x^2 - 2)}{x^2 + 1}\, dx$, f) $\displaystyle\int \sin(2u)\cos(3u)\, du$, g) $\displaystyle\int x^2\sqrt{x + 2}\, dx$, h) $\displaystyle\int \frac{\sec x + \cos x}{3\cos x}\, dx$,

i) $\displaystyle\int \frac{\sin(\sqrt{u})}{\sqrt{u}}\, du$, j) $\displaystyle\int \frac{\sec^2 x}{4 + \tan x}\, dx$, k) $\displaystyle\int e^u(1 + e^u)^{17}\, du$, l) $\displaystyle\int \frac{x^2}{1 + x^6}\, dx$, m) $\displaystyle\int \frac{y}{\sqrt{2y + 1}}\, dy$,

n) $\displaystyle\int \frac{\sin\theta}{\cos^2\theta + 1}\, d\theta$, o) $\displaystyle\int \frac{t}{t + 3}\, dt$, p) $\displaystyle\int \tan x\, dx$, q) $\displaystyle\int \frac{e^z}{4 + e^{2z}}\, dz$, r) $\displaystyle\int \sin(\sin\theta)\cos\theta\, d\theta$,

s) $\displaystyle\int \frac{1}{\sqrt{9 - x^2}}\, dx$, t) $\displaystyle\int \cos(4t)\cos(7t)\, dt$, u) $\displaystyle\int \frac{t}{1 + \sqrt{t}}\, dt$, v) $\displaystyle\int \frac{\sqrt[4]{x}}{1 + \sqrt{x}}\, dx$, w) $\displaystyle\int \frac{\cos^3\theta}{\sin^6\theta}\, d\theta$,

x) $\displaystyle\int \frac{\sin u}{(\cos u + \sin u)^3}\, du$, y) $\displaystyle\int \cos x(-3 + 2\frac{\sin x}{\cos^3 x})\, dx$,

7. Solve each initial-value problem.

a) $\dfrac{dy}{dx} = \dfrac{x^4 - 1}{x^2 + 1}$, $y(1) = \pi/2$, b) $\dfrac{dy}{dt} = \dfrac{1}{25 + 9t^2}$, $y(-5/3) = \pi/30$.

8. a) Find all values of t at which the parametric curve $x = 3t^4 - 3t^2, \quad y = t^3 - 4t$, has i) a horizontal tangent line, ii) a vertical tangent line. b) Show that the parametric curve $x = t^2 - 3t + 5, \quad y =$

$t^3 + t^2 - 10t + 9$ intersects itself at the point (3,1), and find equations for the two tangent lines at the point of intersection. Also find the values of $\frac{d^2y}{dx^2}$ at the point of intersection. c) Eliminate the parameter in the equations of the curve $x = \sin^2 t, \quad y = 4\cos^2 t$, $(0 \le t \le \pi/2)$ sketch the curve, and indicate its orientation. d) Consider the parametric curve given by $x = t$, $y = 1 - t^2, \quad 1 \le t \le 2$. i) Find $\frac{dy}{dx}$. ii) Eliminate the parameter, and sketch that curve including its orientation.

e) The path of a fly is given by the equations of motion

$$x = \frac{\cos t}{2 + sint}, \quad y = 3 + \sin(2t) - 2\sin^2 t, \quad 0 \le t \le 2\pi.$$

a) How high and how low does it fly?
b) How far left and right of the origin does it fly?
c) When is it flying horizontally?
d) When is it flying vertically?