MAC 2313 (Calculus III)
Test 4 Review: This test covers sections 14.4, 14.7, 14.8, 15.1,and 15.5

1. a) Let G be the solid defined by the inequalities: $\sqrt{x^2+y^2} \le z \le 20 - x^2 - y^2$. Find the coordinates of the centroid of G. b) Find the mass and center of gravity of the solid G enclosed by the portion of the sphere $x^2+y^2+z^2=2$ on or above the plane $z=1$ if the density is $\delta = \sqrt{x^2+y^2+z^2}$.

2. a) Set up, but do not evaluate, two iterated integrals equal to the surface integral $\int\int_\sigma y^2 z dS$, where σ is the portion of the cylinder $x^2+z^2=4$ in the first octant between the planes $y=0$, $y=6$, $x=z$, and $x=2z$. b) Consider the parametric surface given by $\mathbf{r}(u,v) = u\overrightarrow{i} + u\cos v\overrightarrow{j} + u\sin v\overrightarrow{k}$ with $0 \le u \le 4$ and $0 \le v \le \pi$. i) Find the area S of σ. ii) Find the mass M of σ if its density is $\delta(x,y,z) = x^2+y^2+z^2$. iii) Evaluate the surface integral $\int\int_\sigma x\sqrt{z}dS$ where σ is the portion of the paraboloid $z = x^2+y^2$ in the first octant between the planes $z=0$ and $z=4$. iv) a) Find an equation for the tangent plane to the parametric surface σ given by: $\overrightarrow{r}(u,v) = 4u\cos v\overrightarrow{i} + u^2\overrightarrow{j} + 3u\sin v\overrightarrow{k}$, at the point P corresponding to $(u,v) = (1,\pi/2)$..

3. Let $\mathbf{F}(x,y,z) = (x^2-2yx)\overrightarrow{i} + (3y^2-2yz)\overrightarrow{j} + (5z^2-2xz)\overrightarrow{k}$. Find div$\mathbf{F}$ and curl$\mathbf{F}$.

4. a) Find parametric equations for the paraboloid $z = x^2+y^2$ in terms of the parameters θ and ϕ, where (ρ,θ,ϕ) are spherical coordinates of a point on the surface. b) Find a parametric representation of the portion of the sphere $x^2+y^2+z^2=9$ on or above the plane $z=2$ in terms of the parameters r and θ, where (r,θ,z) are the cylindrical coordinates of a point on the surface.

5. Find the Jacobian $\partial(x,y)/\partial(u,v)$. a) $u = x^2+y^2$, $v = xy$. b) $u = x^2-y^2$, $v = 2x-y$.

6. Find the Jacobian $\partial(x,y,z)/\partial(u,v,w)$. a) $u = xy$, $v = yz$, $w = x+z$. b) $x = u-uv$, $y = uv-uvw$, $z = uvw$.

7. Evaluate the integral by making an appropriate change of variables.

a) $\int\int_R \frac{\sin(x-y)}{\cos(x+y)}\, dA$, where R is the triangular region enclosed by the lines $y=0$, $y=x$, $x+y=\pi/4$.

b) $\int\int_R e^{\frac{(y-x)}{(y+x)}}\, dA$, where R is the region in the first quadrant enclosed by the trapezoid with vertices (0,1), (1,0),
(0,4), (4,0).

8. Use the transformation $u = xy$, $v = x^2-y^2$ to evaluate $\int\int_R (x^4-y^4)e^{xy}\, dA$, where R is the region in the first quadrant enclosed by the hyperbolas $xy=1$, $xy=3$, $x^2-y^2=3$, $x^2-y^2=4$.

9. Let G be the solid defined by the inequalities: $1-e^x \le y \le 3-e^x$, $\quad 1-y \le 2z \le 2-y$, $\quad y \le e^x \le y+4$.

a) Using the change of variables $u = e^x+y$, $v = y+2z$, $w = e^x-y$, find the Jacobian $\partial(x,y,z)/\partial(u,v,w)$ and express it in terms of u, v, and w. b) Find the volume of G using the change of variables in part a). c) Write down the coordinates of the centroid of G, include for each coordinate the appropriate limits of integration, bu do not evaluate any of the triple integrals involved.