MAC 2313 (Calculus III)
Test 4 Review: This test covers sections 14.4, 14.8, 15.1, 15.2, 15.3, 15.4, 15.5 and 15.6

1. a) Let G be the solid defined by the inequalities: $\sqrt{x^2+y^2} \le z \le 20 - x^2 - y^2$. Find the coordinates of the centroid of G. b) Find the mass and center of gravity of the solid G enclosed by the portion of the sphere $x^2+y^2+z^2 = 2$ on or above the plane $z = 1$ if the density is $\delta = \sqrt{x^2+y^2+z^2}$.

2. a) State the fundamental theorem of line integral. b) Let $F(x,y) = (2xy+x)\overrightarrow{i} + (x^2+2y)\overrightarrow{j}$. b1) Show that F is conservative. b2) Find a potential function φ for F. b3) Evaluate the line integral $\int_{\mathcal{C}} (2xy+x)\,dx + (x^2+2y)\,dy$ along the curve $\mathcal{C}$ parametrized by $\overrightarrow{r}(t) = \sqrt{1+t}\,\overrightarrow{i} + \sin^{-1} t\,\overrightarrow{j}, \quad 0 \le t \le 1$.

3. a) Set up, but do not evaluate, two iterated integrals equal to the surface integral $\int\int_\sigma y^2 z dS$, where σ is the portion of the cylinder $x^2+z^2 = 4$ in the first octant between the planes $y = 0$, $y = 6$, $x = z$, and $x = 2z$. b) Consider the parametric surface given by $\mathbf{r}(u,v) = u\,\overrightarrow{i} + u\cos v\,\overrightarrow{j} + u\sin v\,\overrightarrow{k}$ with $0 \le u \le 4$ and $0 \le v \le \pi$. i) Find the area S of σ. ii) Find the mass M of σ if its density is $\delta(x,y,z) = x^2+y^2+z^2$. iii) Evaluate the surface integral $\int\int_\sigma x\sqrt{z} dS$ where σ is the portion of the paraboloid $z = x^2+y^2$ in the first octant between the planes $z = 0$ and $z = 4$. iv) a) Find an equation for the tangent plane to the parametric surface σ given by: $\overrightarrow{r}(u,v) = 4u\cos v\,\overrightarrow{i} + u^2\,\overrightarrow{j} + 3u\sin v\,\overrightarrow{k}$, at the point P corresponding to $(u,v) = (1,\pi/2)$..

4. Let $F(x,y) = (x^3y + 4e^{-2x})\overrightarrow{i} + (\frac{x^4}{4} + y^2)\overrightarrow{j}$. a) Show that F is conservative. b) Find a potential function φ for F. c) Evaluate the line integral $\int_{\mathcal{C}} (x^3y + 4e^{-2x})\,dx + (\frac{x^4}{4} + y^2)\,dy$ along the curve $\mathcal{C}$ parametrized by $\overrightarrow{r}(t) = \cos^3 t\,\overrightarrow{i} + \sin^3 t\,\overrightarrow{j}, \quad 0 \le t \le \pi$.

5. a) Let $\mathbf{F}(x,y,z) = (x^2 - 2yx)\overrightarrow{i} + (3y^2 - 2yz)\overrightarrow{j} + (5z^2 - 2xz)\overrightarrow{k}$. Find div$\mathbf{F}$ and curl$\mathbf{F}$. Evaluate the line integral $\int_{\mathcal{C}} \text{curl}\mathbf{F} \cdot d\mathbf{r}$, where $\mathcal{C}$ is the triangle with vertices (0,0,2), (0,2,0) and (2,0,0).

6. Let $\mathcal{C}$ be the curve given by $x = t$, $y = 3t^2$, $z = 6t^3$, $\quad 0 \le t \le 1$, and evaluate $\int_{\mathcal{C}} xyz^2\,ds$. b) Evaluate the line integral along $\mathcal{C}$ given by $\mathcal{C}: x = t,\ y = t^2,\ z = 3t^2,\ 0 \le t \le 1$, $\int_{\mathcal{C}} \sqrt{1+30x^2+10y}\,ds$. c) Evaluate $\int_{\mathcal{C}} ydx + zdy - xdz$ along the helix $x = \cos(\pi t), \quad y = \sin(\pi t), \quad z = t$ from the point (1,0,0) to (-1,0,1). d) Find the mass of a thin wire shaped in the form of the curve $x = e^t\cos t$, $y = e^t \sin t$, $(0 \le t \le 1)$ if the density function δ is proportional to the distance to the origin.

7. a) Find parametric equations for the paraboloid $z = x^2+y^2$ in terms of the parameters θ and ϕ, where (ρ,θ,ϕ) are spherical coordinates of a point on the surface. b) Find a parametric representation of the portion of the sphere $x^2+y^2+z^2 = 9$ on or above the plane $z = 2$ in terms of the parameters r and θ, where (r,θ,z) are the cylindrical coordinates of a point on the surface.

8. a) Let $\mathbf{F}(x,y,z) = \sqrt{x^2+y^2}\,\overrightarrow{k}$. Find the flux of $\mathbf{F}$ across σ, where σ is the portion of the cone $\overrightarrow{r}(u,v) = u\cos v\,\overrightarrow{i} + u\sin v\,\overrightarrow{j} + 2u\,\overrightarrow{k}$, with $0 \le u \le \sin v, \quad 0 \le v \le \pi$. b) Let $\mathbf{F}(x,y,z) = x\,\overrightarrow{i} + y\,\overrightarrow{j} + 2z\,\overrightarrow{k}$. Find the flux of $\mathbf{F}$ across σ, where σ is the portion of the paraboloid below the plane $z = y$, oriented by downward unit normals.

9. Let $\mathbf{F}(x,y,z) = (x^3 - e^y)\overrightarrow{i} + (y^3 + \sin z)\overrightarrow{j} + (z^3 - xy)\overrightarrow{k}$. Use the Divergence Theorem to find the flux of $\mathbf{F}$ across σ, where:

a) σ is the boundary of the solid G, bounded above by the sphere $z = \sqrt{4-x^2-y^2}$ and below by the xy-plane, with outward orientation.

b) σ is the boundary of the cylindrical solid enclosed by $x^2+y^2 = 4$, $z = 0$ and $z = 1$ with outward orientation.

10. a) Use Green's theorem to evaluate the line integral $\int_{\mathcal{C}} (4y + \cos(1+e^{\sin x}))\,dx + (2x - \sec^2 y)\,dy$, where $\mathcal{C}$ is the circle $x^2+y^2 = 9$ going from (0,3) to (0,3) counterclockwise.

11. Review the Fundamental Theorem of Line Integral, Green's Theorem and the Divergence Theorem.