Effect of windowing on lithosphere elastic thickness estimates obtained via the coherence method: Results from northern South America

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[1] The effective elastic thickness (Te) of the lithosphere is a parameter that describes the flexural strength of a plate. A method routinely used to quantify this parameter is to calculate the coherence between the two-dimensional gravity and topography spectra. Prior to spectra calculation, data grids must be “windowed” in order to avoid edge effects. We investigated the sensitivity of Te estimates obtained via the coherence method to mirroring, Hanning and multitaper windowing techniques on synthetic data as well as on data from northern South America. These analyses suggest that the choice of windowing technique plays an important role in Te estimates and may result in discrepancies of several kilometers depending on the selected windowing method. Te results from mirrored grids tend to be greater than those from Hanning smoothed or multitapered grids. Results obtained from mirrored grids are likely to be over-estimates. This effect may be due to artificial long wavelengths introduced into the data at the time of mirroring. Coherence estimates obtained from three subareas in northern South America indicate that the average effective elastic thickness is in the range of 29–30 km, according to Hanning and multitaper windowed data. Lateral variations across the study area could not be unequivocally determined from this study. We suggest that the resolution of the coherence method does not permit evaluation of small (i.e., ~5 km), local Te variations. However, the efficiency and robustness of the coherence method in rendering continent-scale estimates of elastic thickness has been confirmed.

INDEX TERMS: 8110 Tectonophysics: Continental tectonics—general (0905); 8159 Tectonophysics: Evolution of the Earth: Rheology—crust and lithosphere; 8160 Tectonophysics: Evolution of the Earth: Rheology—general; 8120 Tectonophysics: Dynamics of lithosphere and mantle—general; KEYWORDS: lithosphere strength, flexural isostasy, north Andes, coherence method, topography and gravity


1. Introduction

[2] Isostasy provides the physical mechanism through which mountain ranges do not “sink” due to the load of their topography. Numerous studies have modeled the isostatic support at mountain ranges by flexure of a thin elastic plate overlying a weak fluid [e.g., Turcotte and Schubert, 1982]. A common parameter used to characterize the stiffness of a lithospheric plate is its effective elastic thickness (Te). This value corresponds to the thickness of an equivalent elastic plate with the same flexural characteristics as the lithosphere. Typically Te ranges from 0 to 105 km [McNutt et al., 1988]. In oceanic regions, Te values roughly coincide with the depth to the 400°C–600°C isotherm [Watts, 1978; Burov and Diament, 1995]. Although in continental regions, Te does not coincide with any physical surface, it is a useful parameter to compare lithosphere strength across different continental plates and to assess the state of flexural compensation at mountain ranges [Karner and Watts, 1983; Whitman, 1994; Watts et al., 1995; Burov and Diament, 1996; Stewart and Watts, 1997; Whitman, 1999]. One of the most widely used techniques to calculate Te is the coherence method [Forsyth, 1985], which evaluates the relationship between the two-dimensional topography and gravity spectra at discrete wavelength intervals. Spectral techniques such as coherence require the data grids to be “windowed” before Fourier transforming to avoid edge effects due to the periodicity assumption in the Fourier transform. The process of data windowing is a necessary step in spectral methods, and it involves a certain degree of distortion of the original data.

[3] Although many studies that used the coherence method have been successful in providing useful Te estimates worldwide, most of these studies assumed that estimates are independent of the windowing technique applied prior to spectra calculation. In this study, we investigated the sensitivity of Te results obtained via the coherence method to the choice of windowing technique.

[4] In order to assess this effect, results from three alternative techniques were compared: (1) mirroring along...
the $x$ and $y$ dimensions; (2) two-dimensional Hanning smoothing; and (3) two-dimensional multitapering. These techniques were applied to computer-generated synthetic data as well as real data from northern South America. The discrepancies among results obtained from these three windowing techniques are presented, as well as comments on the source of such discrepancies.

2. Modeling Lithospheric Flexure

[5] Several studies have modeled the isostatic support of mountain belts by flexure of a thin elastic plate overlying a weak fluid [Turcotte and Schubert, 1982]. The deflection, $w$, of an infinite elastic plate loaded by topography, $h$, is governed by the following differential equation:

$$\nabla^2 (D \nabla^2 w) + \Delta pgw = \rho gh,$$

where $D$ is the plate’s flexural rigidity, $\rho_i$ is the density of the topography (crustal density), and $\Delta \rho$ is the density contrast between the compensating root and the surrounding mantle rock [Turcotte and Schubert, 1982].

[6] The flexural rigidity, $D$, is a measure of the plate’s stiffness and is related to the mechanical properties of the plate by

$$D = \frac{Et_0^2}{12(1-\nu^2)},$$

where $E$ is the Young’s modulus, $\nu$ is the Poisson’s ratio, and $t_0$ is the effective elastic thickness.

[7] $t_0$ is usually measured by analyzing the geophysical expression of interfaces embedded within the lithosphere. Commonly, the Bouguer gravity anomaly associated with deflection of the Moho is modeled to determine $t_0$. By comparing the wavelengths of Moho deflection with theoretical flexural models, one can predict which of these wavelengths are associated with local isostasy and which are due to regional plate flexure.

3. Methods to Calculate $t_0$

[8] Most methods that calculate the effective elastic thickness of the lithosphere are based on the spatial or spectral relationship between topography and gravity, analyzed from profiles or maps. At the profile level, one simple method consists of calculating the deflection of the Moho due to the load of surface topography. Because the characteristics of Moho deflection are a function of the plate’s flexural rigidity and hence of the lithosphere elastic thickness, this deflection can be evaluated for a series of theoretical $t_0$ values. The gravity response due to the combined effects of surface topography and Moho deflection can be compared with the observed anomaly. The best fitting model is commonly chosen by error minimization.

[9] At the two-dimensional, i.e., map, level the relationship between topography and gravity is best evaluated in the frequency or Fourier domain. For this purpose, the topography and gravity spectra are analyzed at discrete wavelength intervals. The transfer function that describes the relationship between the spectra is compared with transfer functions calculated for a series of theoretical models, until a good fit is achieved. The most widely used spectral techniques of $t_0$ estimation are the admittance and the coherence methods, described next.

[10] The admittance method [Dorman and Lewis, 1970; Banks et al., 1977] is a pioneer technique for spectral-domain estimation of flexural rigidity from topography and gravity maps. A number of studies of isostasy have utilized this method for studies of continental $t_0$ [Kogan and McNutt, 1987; McNutt and Kogan, 1987; McKenzie and Fairhead, 1997]. The admittance method evaluates the isostatic response function between two-dimensional topography and gravity spectra and is calculated from

$$Q(k) = \frac{\langle BH^* \rangle}{\langle HH^* \rangle},$$

where $H$ and $B$ represent the Fourier transforms of the topography and gravity, the angle brackets represent averaging over discrete wave number bands, and the asterisks indicate the complex conjugate. In interpreting admittance, investigators commonly assume that isostatic support is provided by elastic plates that are entirely loaded from the top. This assumption works well in areas where loading can be considered to be mostly due to surface topography. However, it ignores loading due to subsurface loads such as buried high-density intrusive bodies, and therefore tends to under-estimate elastic thickness. To avoid the top-loading assumption implicit in the admittance method, the coherence method [Forsyth, 1985] allows consideration of both top and bottom loading.

4. Coherence Method of $t_0$ Estimation

[11] Coherence is the square of the correlation coefficient between two fields and is usually wavelength dependent. It represents the fraction of the power of one field that can be predicted by a linear transfer function applied to the second field. For studies of elastic thickness, the coherence between the observed topography and gravity spectra is compared to the calculated coherence functions of theoretical models with given flexural rigidities [Forsyth, 1985; Bechtel et al., 1987]. If both surface and subsurface loads are present, short wavelength topography is expected to be incoherent with the gravity field; long wavelength topography is coherent. The transition from coherent to incoherent gives an indication of the wavelength where loads start to be regionally supported, and hence this transition contains information on the elastic thickness of the lithosphere [Forsyth, 1985].

[12] The coherence $\gamma^2$ between gravity and topography can be calculated from

$$\gamma^2(k) = \frac{\langle BH \rangle^2}{\langle HH^* \rangle \langle BB^* \rangle},$$

where

$$k = \sqrt{k_x^2 + k_y^2}$$

is the magnitude of the two-dimensional wave number, $B$ and $H$ are the two-dimensional Fourier transforms of the Bouguer gravity and topography, the asterisk indicates
complex conjugation, and the angle brackets indicate averaging over discrete log-spaced wave number annuli [Forsyth, 1985].

[15] In order to avoid bias by noise, a modified coherence function

$$\gamma^2(k) = \frac{(n\gamma^2_o - 1)}{(n - 1)}$$ (6)

should be used, where $n$ represents the number of Fourier coefficients in a given wave number annulus [Munk and Cartwright, 1966]. For all calculations in this study, averages were obtained over 20 discrete wave number annuli.

[14] Assuming that subsurface loading is due to initial relief at the Moho and that surface and subsurface loading are uncorrelated, the observed coherence calculated from equations (4) and (6) can be compared to the predicted coherence from

$$\gamma^2 = \frac{(H_f W_f + H_b W_b)^2}{(H_f^2 + H_b^2)(W_f^2 + W_b^2)},$$ (7)

where $H_f$ is the amplitude of surface topography due to surface loads, $W_f$ is the amplitude of Moho deflection due to surface topography, $H_b$ is the amplitude of the surface topography due to loading on the Moho, and $W_b$ is the amplitude of Moho deflection due to loading on the Moho. The relationships among these expressions are given by

$$W_T(k) = -\rho H_T(k)/\Delta \rho$$ (8)

$$W_g(k) = -\rho H_g(k)\Phi/\Delta \rho$$ (9)

$$|H_o| = f\rho |H_T|/\xi \Delta \rho$$ (10)

$$\xi = 1 + Dk^4/\Delta \rho g$$ (11)

$$\Phi = 1 + Dk^4/\rho g.$$ (12)

[15] The coherence method allows input of a top-to-bottom loading ratio $f$ (equation (10)) for each Fourier component, which can be based on the observed amplitudes of gravity and topography data [Bechtel et al., 1987], and facilitates consideration of buried loads. For simplicity, in all analyses in this paper $f$ was held constant at 1, since the coherence is not too sensitive to changes in $f$ [Forsyth, 1985].

[16] Following [Forsyth, 1985], predicted coherence was calculated from equation (7) at 1 km intervals, except for synthetic data derived from a 10 km thick plate, where 100 m steps were used. Each predicted coherence function was then compared to the observed coherence by computing the L one norm, defined as

$$\sum_{i=1}^n \left| \frac{\gamma^2_{(obs)} - \gamma^2_{(calc)}}{\Delta \gamma^2_i} \right| ,$$ (13)

where $\gamma^2_{(obs)}$ and $\gamma^2_{(calc)}$ are the observed and predicted coherence of the $i$th wave number average, and $\Delta \gamma^2_i$ is the standard error of each coherence estimate, calculated from [Bechtel et al., 1987]

$$\Delta \gamma^2 = (1 - \gamma^2_o)(2\gamma^2_o/n)^{1/2},$$ (14)

resulting in a “U-shaped” curve of L-one norm errors as a function of $Te$. The best fitting elastic thickness was the one whose predicted coherence function returned the smallest L one norm error. The L one norm is a robust method for error estimation of data that contain a few spurious points [Menke, 1989]. The L one norm was therefore preferred over the more common L 2 norm for $Te$ estimation, to minimize the effect of outliers that fall relatively far from the characteristic wavelength, which is the portion of the coherence curve where $Te$ is mainly represented [Forsyth, 1985].

5. Spectral Analysis and Data Windowing

[17] The spectrum of a one- or multidimensional signal is obtained by calculating its Fourier transform, which is easily implemented in most computer applications. An undesired effect during Fourier transformation is spectral leakage, which results from sharp discontinuities in the data. All Fourier methods presume periodicity, so if the ends of the profiles or edges of the study area are not at the same level, there is an implied discontinuity. This effect results in smearing of frequencies onto parts of the spectrum where they do not belong, creating a false representation of the frequencies contained in the signal. In order to avoid these edge effects, data series or grids are “windowed” by altering the data shape in a way that avoids sharp discontinuities while maintaining the original frequency distribution (Figure 1).

[18] We tested the sensitivity of $Te$ estimates obtained by the coherence method [Forsyth, 1985] to three different windowing techniques: Mirroring, Hanning smoothing and multitapering. Mirroring and Hanning smoothing are widely used windowing methods in coherence studies. The multitaper technique is a more novel method and thus its usefulness in studies of isostasy has only been recently realized [Scheirer et al., 1995; McKenzie and Fairhead, 1997; Simons et al., 2000]. Figure 1 illustrates the effect of these three windowing methods on a sample profile across the Colombian Andes.

5.1. Mirroring

[19] Mirroring is the addition of mirror images of the data along the ends and is equivalent to applying a cosine transform to the data (Figure 1b). In the two-dimensional case, mirror images of the data grid are added along the $x$ and $y$ directions, resulting in a grid with 4 times the number of elements of the original one. This technique makes the data grids periodic, an assumption of Fourier methods. However, if the profiles or grids are not flat at the ends, it introduces discontinuities in the implied slope of the fields [Diament, 1985] and therefore does not completely eliminate the problem of spectral leakage. In addition, phase information is lost, eliminating the possibility of recognizing phase shifts between gravity and topography, biasing the coherence function. The undesired shift in coherence functions obtained from mirrored data was recognized by Lowry and Smith [1994], who proposed that mirroring introduces incoherent noise into the spectra. They calculated $Te$ over an
area in western United States based on maximum entropy methods, and observed that coherence functions of mirrored data are shifted toward smaller wave numbers with respect to unmirrored data. In the discussion section we shall elaborate on the importance of this shift and its effect on final $Te$ estimates.

5.2. Hanning

[20] Windows with values that gradually damp the end of the signal to a common mean value (often zero) in a way that avoids sharp discontinuities are useful to decrease frequency leakage. Examples of these windows are Welch, Parzen, Bartlett, and Hanning, which although slightly different in shape offer very similar leakage resistance [Press et al., 1986]. For the purpose of comparing the effect of several windowing techniques on coherence estimates, the Hanning window was selected, due to its common use and simple implementation. The Hanning window is a cosine-shaped taper with values that gradually extend from zero at the edges to unity at the center (Figure 1c). This configuration...
applies more weight to the central part of the input signal while reducing the effect of the borders.

[21] Although effective in decreasing frequency leakage, the Hanning window also has the disadvantage of discarding significant portions of data near the edges by forcing convergence to zero (Figure 1d). This data loss in the spatial domain is also reflected in loss of information content in the frequency domain, decreasing the variance of the spectrum. Such a tradeoff between frequency leakage resistance and conservation of data variance is a well-known issue in signal processing, and has been discussed extensively [Press et al., 1986; Park et al., 1987].

5.3. Multitaper Method

[22] An ideal taper is one that offers the greatest leakage resistance while discarding the least amount of data. The multitaper method is a technique that closely approximates the effect of an ideal taper by extracting the quasi-true spectra of signals. Instead of filtering the data through a single taper such as a Hanning window, the input signal is multiplied by a series of orthogonal “eigentapers” of different shapes [Thomson, 1982; Park et al., 1987; Simons et al., 2000].

[23] For an \( N \) point time series, an eigentaper with “time bandwidth product” \( P = NW \) concentrates spectral energy in frequency bands of width \( 2W \). Eigentapers can be constructed for orders \( k = 0, 1, 2, \ldots, N-1 \) (Figure 1e). However, as \( k \) increases, their leakage resistance decreases, and in consequence, only the lowest order eigentapers are useful for minimizing spectral leakage [Park et al., 1987].

[24] One important characteristic of the multitaper method is the mutual orthogonality of the eigentapers. Due to this property, the information lost by the first taper is regained by the next orthogonal taper, which in turn is balanced by the next taper and so on. The net result is a greater conservation of information, relative to single-taper filters such as Hanning windows (Figure 1f).

[25] After tapering the data with each eigentaper, the discrete Fourier transform is calculated separately resulting in an equal number of “eigenspectra.” The final spectrum is obtained either by a simple combination of the eigenspectra or as a weighted sum of them, depending on the implementation. A thorough discussion of multitaper spectral analysis applied to studies of isostasy is given by Simons et al. [2000].

[26] We used the two-dimensional implementation of the multitaper method [Thomson, 1982] described by Hansen [1997]. We averaged the eight lowest-order eigentapers only (using 4 as the time-bandwidth product), as these have eigenvalues close to unity and are thus useful for minimizing spectral leakage [Park et al., 1987]. Once the spectrum is calculated by this method, it can be directly imported into the coherence function described in equation (4).

6. Results From Synthetic Data

[27] To investigate the accuracy of the coherence function calculated with different windowing methods, the effect of windowing was tested on computer-generated topography and gravity grids.

[28] The first step was the generation of the initial loads acting on the plate. Two synthetic surfaces were generated: the first surface represented the initial load acting at the top of the plate, \( H_p \), and the second one represented the initial load acting at the bottom of the plate, \( W_p \). These surfaces were produced according to the method outlined by Turcotte [1992]. For each surface, a grid of Gaussian white noise was created by building a two-dimensional matrix of random numbers, and its two-dimensional Fourier transform was calculated. The power spectrum of each surface was stretched to a new slope \( \beta = 7 - 2D_2 \), and its inverse Fourier transform was calculated. This resulted in two topography-like surfaces with the specified fractal dimension. Following Turcotte [1992], the surfaces were generated using a fractal dimension \( D_2 \) of 2.2. Each grid contained \( 256 \times 256 \) elements, with cell sizes of 10 or 20 km.

[29] The second step consisted of loading the surface and subsurface loads onto a plate of determined flexural rigidity, and then calculating the final amplitudes of the surface topography, \( H \), and of the Moho, \( W \). Following Forsyth [1985], this was accomplished by using

\[
H = H_f \left\{ \frac{\Delta \rho_k}{\rho_0 + \Delta \rho} \right\} - W_f \left\{ \frac{\Delta \rho}{\Delta \rho + \rho_0} \right\},
\]

\[
W = -H_f \left\{ \frac{\rho_0}{\rho_0 + \Delta \rho} \right\} - W_f \left\{ \frac{\rho_0}{\rho_0 + \Delta \rho} \right\},
\]

[30] The third step was calculation of the Bouguer gravity anomaly. This was done separately for the surface topography and the Moho. The Bouguer gravity due to the surface topography was calculated from the wave number or Fourier domain solution of the flexure equation,

\[
B_t(k) = -2\pi G\rho_1 \frac{e^{-ikz}}{1 + \frac{k^2}{\Delta \rho}} H(k),
\]

where \( G \) is the universal gravitational constant, and the remaining variables are as defined earlier.

[31] The Bouguer gravity due to the deflected Moho was calculated from

\[
B_m(k) = 2\pi G\rho_0 W(k) \left[ \exp(-kz_2) - \exp(-kz_1)/(1 + Dk^4/\Delta \rho) \right],
\]

where \( z_i \) is the mean depth to the subsurface load and \( z_1 \) is the mean depth of compensation. The combined Bouguer anomaly was calculated by adding the contributions of the topography (equation (16)) and the contribution of the Moho (equation (17)). The final step was extraction of internal subsets of the topography and gravity away from the edges, to ensure that the gravity field contained information from structures outside the topography boundaries, as in the real world.

[32] Using the above procedure, 20 topography-gravity sets were generated for each elastic plate thickness of 10, 30, and 90 km, resulting in a total of 60 data sets. For each of these sets, coherence was calculated using the mirroring, Hanning, and multitaper windowing methods. Best fitting elastic thickness values were estimated from the calculated coherence results by L one norm minimization (Figure 2). The elastic parameters used in the models are listed in Table 1.
Table 2 summarizes the results from synthetic data, including the means and standard deviations of the 20 trials for each windowing method and input elastic thickness. For the 10 km plate model, results from mirrored grids overestimated $Te$ by a factor of 4, while Hanning and multitaper overestimated it by factors of 2 and 1.7 respectively. For the 30 km plate, multitapering returned the most accurate estimates, with an average of 31.0 km. Average estimates from mirrored grids were 2.1 times the input thickness, and Hanning estimates were in average 1.3 times greater than the true value. For the 90 km plate model, mirroring provided estimates 1.2 times greater and Hanning as well as multitapering underestimated the true value by factors of 0.93 and 0.82, respectively.

The standard deviations shown in Table 2 are a measure of statistical error rather than systematic error. In other words, a very small standard deviation only indicates that the estimator was able to reproduce the same $Te$ for repeated trials, regardless of what the underlying $Te$ really is. The deviations calculated for the trials suggest that for weak ($Te = 10$ km) and moderately strong ($Te = 30$ km) plates, the multitapering method provided the most repeatable results, returning estimates that ranged within 43% and 26% of the average. For strong ($Te = 90$ km) plates, all methods produced estimates within one third of the average.

Table 2. Elastic Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v$ (Poisson's ratio)</td>
<td>0.25</td>
</tr>
<tr>
<td>$E$ (Young's modulus)</td>
<td>$1 \times 10^{-11}$ $\text{m}^2/\text{kg} \cdot \text{s}^2$</td>
</tr>
<tr>
<td>$\rho_c$ (mean crustal density)</td>
<td>2670 $\text{kg}/\text{m}^3$</td>
</tr>
<tr>
<td>$\Delta \rho$ (crust-mantle density contrast)</td>
<td>680 $\text{kg}/\text{m}^3$</td>
</tr>
<tr>
<td>$\rho_t$ (subsurface load density)</td>
<td>3170 $\text{kg}/\text{m}^3$</td>
</tr>
<tr>
<td>$t_c$ (average crustal thickness)</td>
<td>35 km</td>
</tr>
<tr>
<td>$z_c$ (mean depth of compensation)</td>
<td>35 km</td>
</tr>
<tr>
<td>$z_l$ (mean depth of subsurface load)</td>
<td>18 km</td>
</tr>
<tr>
<td>$f$ (top-to-bottom loading ratio)</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 2. Coherence results for sample individual trials obtained from synthetic topography. Top, middle, and bottom rows show coherence estimates obtained for elastic plates of 10, 30 and 90 km $Te$ respectively. Left, middle, and right columns contain results from mirrored, Hanning-smoothed and multitapered grids. In each plot, dots are coherence averages and vertical bars are plus or minus one standard error. Continuous lines and numbers next to them are best fitting function and corresponding $Te$ estimate.
Table 2. Results From Synthetic Data

<table>
<thead>
<tr>
<th>Method</th>
<th>$Te = 10$ km</th>
<th>$Te = 30$ km</th>
<th>$Te = 90$ km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirroring</td>
<td>41.6 ± 23.0</td>
<td>62.2 ± 46.3</td>
<td>110 ± 31</td>
</tr>
<tr>
<td>Hanning</td>
<td>20.1 ± 10.6</td>
<td>39.7 ± 33.4</td>
<td>84 ± 28</td>
</tr>
<tr>
<td>Multitaper</td>
<td>17.1 ± 7.3</td>
<td>31.0 ± 8.1</td>
<td>73 ± 22</td>
</tr>
</tbody>
</table>

Hanning, in agreement with Scheirer et al. [1995] and Simons et al. [2000] and (2) coherence results from mirrored data tend to overestimate $Te$.

7. Test With Real Data: Northern South America

[36] The effect of windowing on coherence estimates was also tested on gravity and topography data from northern South America, in an area that encompasses parts of both Colombia and Venezuela. A brief description of the tectonic setting of this area and the characteristics of the data follows.

[37] The study area extends for 1600 km in the east-west direction and about 1000 km in the north south (Figure 3a). The 1000-m contour outlines significant topographic features in this region, which include the Andean ranges and the highlands of the Guyana Shield. The Eastern Cordillera (Cordillera Oriental) is a broad NNE-SSW trending range that reaches elevations up to 4 km, and represents the easternmost branch of the Colombian Andes. The Eastern Cordillera takes a sharp west turn, and continues due NNW with the name of Santander Massif, which narrows rapidly and connects with the NNE-trending Sierra de Perijá. The northeastern termination of the Andes is represented by the Venezuelan Andes (Mérida Andes), which run northeastward across western Venezuela.

[38] Rocks ranging from Precambrian to Pleistocene crop out in and around the study area, suggesting a complex tectonic history. During the Triassic and Early Jurassic the region was extensively rifted. The locus of extension corresponds with today’s high topography, suggesting tectonic inversion of normal faults. This rifting episode appears to be the last thermal event affecting the north Andean lithosphere before the Neogene and may represent the start of its thermal age. A “Pre-Andean” tectonic event is manifested in the north Andes by a regional unconformity dated Middle Eocene, and recognized in the sub-Andean foreland basins. Uplift of the eastern portion of the north Andes seems to have started during the middle Miocene as well as the three subareas where $Te$ was calculated are shown in Figure 3c. Two large anomaly lows are identifiable: a NNE-SSW trending gravity low that extends along the axis of the Eastern Cordillera of Colombia and an E-W elliptical gravity low near the northeastern corner of the study area, west of Trinidad. Gravity anomaly highs (greater than zero mGal) are found in central Venezuela and the northwestern corner of the study area.

[40] Public domain topography and gravity data were used for coherence calculation. The original topographic data were the GTOPO30 30 see global digital elevation model available from the USGS EROS Data Center [Gesch et al., 1999].

[41] Point gravity data were taken from the South America gravity database available from Hittelman et al. [1994]. The distribution of these points allowed good coverage of the entire study area (Figure 3b). A total of 30,078 gravity points from this database fall within the limits of the study area. This number of stations resulted in an average point density of 1.7 stations for every 10 $\times$ 10 km square cell. Prior to generation of the final Bouguer gravity grid, gravity points that generated “bull’s-eye” effects were identified, and removed from the database. No terrain corrections were applied to the gravity data. The Bouguer gravity points were gridded every 10 km using the method of splines in tension [Smith and Wessel, 1990], using a relatively high tension coefficient to produce a smooth surface (Figure 3c).

[42] The study area was selected to cover a fairly heterogeneous tectonic region where lateral changes in elastic thickness could be expected, based on the $Te$ map of northern South America by Stewart and Watts [1997]. The region was divided into overlapping square subareas A, B and C, 1000 km by 1000 km each, where independent $Te$ estimates were obtained. Variations among these estimates were expected to give an indication of the degree of lateral change in elastic thickness. For simplicity, we chose not to split the area according to tectonic features or topography trends. However, area A is mostly representative of the mountain region, area C is mostly cratonic, and area B is a transition between the other two domains.

[43] Bouguer gravity anomalies in northern South America as well as the three subareas where $Te$ was calculated are shown in Figure 3c. Two large anomaly lows are identifiable: a NNE-SSW trending gravity low that extends along the eastern Cordillera of Colombia and an E-W elliptical gravity low near the northeastern corner of the study area, west of Trinidad. Gravity anomaly highs (greater than zero mGal) are found in central Venezuela and the northwestern corner of the study area.

In addition,
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a) Map showing geographic regions and areas labeled as Area A, Area B, and Area C.

b) Map with geographic areas A, B, and C indicated.

c) Map depicting variations in mGal values across the same geographic regions.
a well-defined east-west trending gravity low that extends for several hundreds of kilometers in northeastern Venezuela near the Orinoco river delta has no correspondence with major topographic features of comparable extent. These observations suggest departures from Airy isostasy.

9. Te Results for Northern South America

Coherence plots for the entire study area and for each of the three subareas A, B, and C are shown in Figures 4 and 5. The resulting Te estimates are summarized in Table 3. As demonstrated with synthetic data, the choice of windowing affects the Te estimates.

Coherence results from mirrored areas A and B are 52 and 34 km, consistently greater than estimates obtained from Hanning and multitapering. For mirrored area C, an estimate could not be obtained because the L one norm error function did not converge to a minimum (Figure 5). As demonstrated in the first part of this manuscript, coherence functions from mirrored data may lead to overestimation of the true elastic thickness. We thus believe that the results from mirrored grids exaggerate the actual plate strength.

Results from 2-D Hanning and 2-D multitapering for areas A, B, and C range between 24 and 37 km, suggesting a moderately strong lithosphere. Although results from the Hanning window suggest that the strongest lithosphere corresponds to area C (Guyana craton), Te values obtained from multitapered data are greater under area A (Andean ranges). The analyses with synthetic data shows that the standard deviations of these two windowing methods may

Figure 4. Coherence results obtained for the entire study area, from (top) mirrored, (middle) Hanning, and (bottom) multitapered grids. Dots represent coherence averages and vertical bars indicate plus or minus one standard error. Continuous lines and numbers show best fitting coherence function and corresponding Te.
be as high as 84% of the estimate. On the basis of this observation, we conclude that small (i.e., <5 km), local $T_e$ variations are difficult to resolve with the two-dimensional coherence implementation. Accordingly, the differences observed across the three areas are more likely to represent the scatter intrinsic to these methods rather than actual elastic thickness changes.

[48] Averages obtained for the three areas using Hanning and multitapering were 29.3 and 30.3 km, close to the 33 and 35 km estimates obtained for the entire area as a single data set. We regard these values as representative of the northern South America lithosphere.

10. Discussion and Conclusions

[49] Unexpectedly large $T_e$ estimates have been obtained with the coherence method over diverse tectonic provinces such as old shields [McKenzie and Fairhead, 1997] and continental rifts [Bechtel et al., 1987]. McKenzie and Fairhead [1997] have discussed the source of this effect and concluded that Forsyth's [1985] method provides only upper bounds rather than estimates, unless the free-air gravity instead of the Bouguer anomaly is employed. Although McKenzie and Fairhead [1997] suggested that the choice of windowing technique may have an effect in final $T_e$ estimates, they argue that over-estimation of elastic thickness is mainly due to the use of the Bouguer gravity. The results presented here suggest that the windowing technique alone is sufficient to explain overestimates of elastic thickness obtained with the coherence method. In particular, the mirroring method of windowing consistently exaggerates the elastic thickness.

[50] The effect of mirroring has been discussed by Lowry and Smith [1994] and McKenzie and Fairhead [1997], who agree that this type of windowing introduces noise into the coherence function manifested at long wavelengths. This study built upon their suggestion of mirroring-induced noise, and documents consistent overestimation of elastic thickness by mirrored synthetic and real data. Results from synthetic data indicate that estimates from mirrored grids may be up to a factor of 4 greater than the true elastic

Table 3. Summary of $T_e$ Estimates for Northern South America

<table>
<thead>
<tr>
<th>Method</th>
<th>Area A</th>
<th>Area B</th>
<th>Area C</th>
<th>Average of A, B, and C</th>
<th>Whole Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirroring</td>
<td>52</td>
<td>34</td>
<td>unresolved</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>2-D Hanning</td>
<td>27</td>
<td>24</td>
<td>37</td>
<td>29</td>
<td>33</td>
</tr>
<tr>
<td>2-D multitapering</td>
<td>34</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>35</td>
</tr>
</tbody>
</table>

In units of km.
thickness. We suggest that this bias has been overlooked in previous coherence studies.

The spurious effect of mirroring can be demonstrated in the space domain. When topography and gravity signals are mirrored, artificial wavelengths are introduced by enlarging the data set. If a topographic feature that is coherent with its gravity anomaly is artificially folded onto a longer wavelength, the coherence function will also be shifted, leading to over-estimation of elastic thickness. This effect is illustrated in Figure 6 for a sample topography and gravity profile across northern South America. The topography feature at the edge of the data and its corresponding gravity anomaly are merged with their mirror images, resulting in an artificial mountain range and a corresponding gravity low, twice as large as the input ones. If in the original data these signals are coherent, in the mirrored data this correspondence will now be shifted toward longer wavelengths. Although this effect is illustrated for features near the edge of the data set, it affects the entire signal. The ultimate result is to shift the transition from coherent to incoherent anomalies toward longer wavelengths causing overestimation of the elastic thickness. In addition, phase information is lost during mirroring eliminating the possibility of recognizing phase shifts between topography and gravity. Many studies that have calculated coherence from mirrored data may have suffered this bias, and conflicting estimates on the Australian lithosphere support this statement. A recent reevaluation of the Australian lithosphere strength using multitaper windowing by Simons et al. [2000] suggests that some of the Te results of Zuber et al. [1989] may have been contaminated by the distorting effect of mirroring.

The efficiency of the multitaper method in coherence estimates suggested by Scheirer et al. [1995] and Simons et al. [2000] was tested and confirmed by this study. In the analyses with synthetic data, the multitaper method rendered more accurate results than Hanning and mirroring, and it should be considered as a more desirable alternative due to its efficiency in handling frequency leakage and spectral variance [Thomson, 1982]. Moreover, the two-dimensional implementation is straightforward [Hanssen, 1997], and does not involve much more computational resources than conventional windowing methods.

Mapping lateral variations of Te with the two-dimensional coherence method is an ambitious task. This study attempted to determine lateral changes within the study area by dividing it into overlapping subareas over which independent Te estimates were obtained. The differences mapped across adjacent subareas in northern South America from multitaper windowing were not greater than 6 km (Table 2). These differences are smaller than the standard deviations obtained for a synthetic 30 km thick plate.

Figure 6. Diagram showing the effect of mirroring on topography and gravity profiles. A topographic feature at the edge of the profile is merged with its mirror image, resulting in an artificial mountain twice as wide. If the original topographic feature is coherent with its gravity anomaly, the coherence function will shift toward longer wavelengths resulting in overestimation of elastic thickness.
Therefore, although lateral $Te$ variations across these subareas may exist, they could not be unequivocally resolved. [55] Coherence results from multitapered topography and gravity indicate that the effective elastic thickness of the lithosphere beneath northern South America lies around 30 km, according to average estimates obtained over overlapping subareas as well as estimates obtained over the entire area as a single grid.

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