# MET3103 and 5105

```
INSOL.m
function [lat, I_noon, I_ave, h_dl, TBB] = INSOL(D,I0)
%Computes as arrays of values as a function of latitude:
%     lat    -- Latitude every 5 deg from South to North Pole
%     I_noon -- Noon insolation (W m^-2)
%     I_ave  -- Dayly averge isdolation (W m^-2)
%     h_dl   -- Hours of daylight (h)
%     TBB    -- Blackbody temerature (zero Albedo)
SB_const = 5.67e-8;         %Set Stefan-Boltzmann constant
lat = -90:5:90;              %Make array of latitudes
CS_Lat = cos(pi*lat/180);    %Make array of cosnes of latitude
SN_Lat = sin(pi*lat/180);    %Make array of sines of latitude
CS_D = cos(pi*D/180);        %Make cosine of declination
SN_D = sin(pi*D/180);        %Make sine of decination
CS_A0 = -SN_Lat.*SN_D./(CS_Lat.*CS_D);   %Compute cosine of sun's LHA at
                                         % sunset
A0 = acos(max( min(CS_A0,1.0),-1.0));    %Compute sunset LHA in radians
SN_A0 = max(sin(A0),0.0);                %Compute sine of LHZ, >= 0
I_noon =  ...                            %Compute noon insolations
       max(I0*(SN_Lat.*SN_D + CS_Lat.*CS_D),0);
I_ave  = ...                             %Compute average insolation
      (I0/pi)*(SN_Lat.*SN_D.*A0 + CS_Lat.*CS_D.*SN_A0);
h_dl = 24*A0/pi;                         %Compute hours of daylight
TBB = (I_ave/SB_const).^0.25;            %Compute blackbody temperature
```

Lecture 04
Climate Models
5 February 2018

## We Humans Try to Understand the World by Building Models

- **Some models are conceptual: "The Sun rises in the east and sets in the west."**
- **Some are physical, like the 0.055 scale Boeing 777X in the wind tunnel at Farnborough in the UK**
- **Others are mathematical**
- **We can scale small physical models to represent full-size systems**
- **Models of all kinds are useful for understanding, prediction, and testing ideas.**

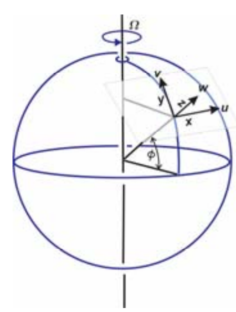
**Rotating Tank Models are Useful (and Fun) for Understanding Climate**


## The Most Powerful Models Are Often Mathematical

- Newtonian Paradigm: Physical Laws Expressed as Differential Equations
- Examples:
  - Classical Mechanics
  - Electricity & Magnetism
  - Thermodynamics
  - Relativity
  - Quantum Mechanics
  - Subatomic Particles
- Differential equations express relations between derivatives
- Theoreticians make predictions.
- Experimentalists invent ways to test models based upon these predictions
- Karl Popper: A <u>scientific</u> theory must be ***Falsifiable*** : If comparison with experimental results comes out wrong, we may be forced to discard or change it.

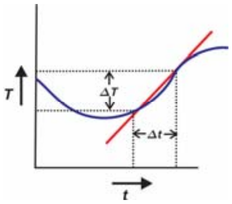
Much of our understanding and ability to predict climate comes from numerical GCMs (Global Climate Models, or formerly, General Circulation Models )

**We actually write models on a spherical Earth, but for simplicity we'll work in a tangent plane Cartesian coordinate system**




- **The Earth rotates counter clockwise with angular velocity Ω**
- **The plane is tangent to the Earth at latitude *φ***
- **The *x* coordinate points east, and the *u* component of the wind blows eastward (Westerly)**
- **The *y* coordinate points north and the *v* component of the wind blows north ward (Southerly)**
- **The *z* coordinate points upward, and the *w* component of the wind blows upward (Downerly???)**

## A Little Calculus

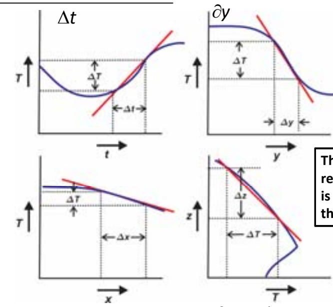



- Derivatives express rates of change of functions
- Consider temperature, $T(t, x, y, z)$, warming after sunrise
- The derivative of temperature with respect to time is a small amount of warming divided by the small amount of time it took
- In calculus we go to the limit of an infinitesimal warming during an infinitesimal time

$$\frac{\partial T(t,x,y,z)}{\partial t} \simeq \frac{T(t+\Delta t,x,y,z)-T(t,x,y,z)}{\Delta t} = \frac{\Delta T}{\Delta t}$$

### Partial Derivatives Express Rates of Change with Respect to Different Variables

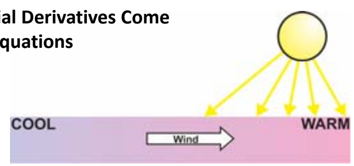
$$\frac{\partial T}{\partial t} = \frac{T(t+\Delta t,x,y,z)-T(t,x,y,z)}{\Delta t} \qquad \frac{\partial T}{\partial y} = \frac{T(t,x,y+\Delta y,z)-T(t,x,y,z)}{\Delta y}$$




The derivative with respect to any variable is calculated keeping the others constant

$$\frac{\partial T}{\partial x} = \frac{T(t,x+\Delta x,y,z)-T(t,x,y,z)}{\Delta x} \qquad \frac{\partial T}{\partial z} = \frac{T(t,x,y,z+\Delta z)-T(t,x,y,z)}{\Delta z}$$

### How Partial Derivatives Come Into the Equations




- If the wind blows from the cool north to the warm south, it brings lower temperatures
- Solar heating (insolation) tends to warm the air
- So that (assuming no pressure change) the equation for the temperature is:

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial y} + Q$$

- The first term on the right is *Advection* of temperature by the wind
- The last term on the right is (net) heating

## The “Complete” Navier-Stokes Equations

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} + fv - \frac{1}{\rho}\frac{\partial p}{\partial x} - F_x \quad \text{x momentum}$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - fv - \frac{1}{\rho}\frac{\partial p}{\partial y} - F_y \quad \text{y momentum}$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z} - p\left(\frac{\partial \rho^{-1}}{\partial t} + u\frac{\partial \rho^{-1}}{\partial x} + v\frac{\partial \rho^{-1}}{\partial y} + w\frac{\partial \rho^{-1}}{\partial z}\right) + Q \quad \text{thermodynamic energy}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial \rho w}{\partial z} = 0 \quad \text{mass continuity}$$

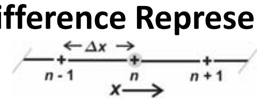
$$\frac{\partial p}{\partial z} = -g\rho \quad \text{hydrostatic law}$$

$$\frac{p}{\rho} = RT \quad \text{gas law}$$

Plus more equations for moisture, rainwater, $CO_2$, etc.

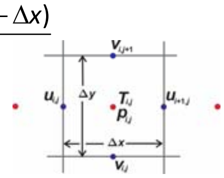
- The first three equations are *“Prognostic”* because they contain time derivatives
- The last three are *“Diagnostic,”* because they do not
- The last terms in the Prognostic equations are forcings that represent the *“Physics”* of the model
- The relationships between wind and pressure are called the *“Dynamics”* in this context

## Finite-Difference Representation




- In GCMs we tabulate the State Variables on a grid such that ***x* = *n*Δx, *y* = *m*Δy**, etc
- Thus, for example, advection of ***T*** by the ***u*** wind component is:

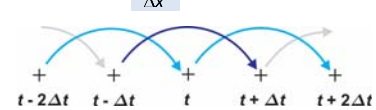
$$u\frac{\partial T}{\partial x} \simeq u(x)\frac{T(x+\Delta x)-T(x-\Delta x)}{2\Delta x}$$




### We use finite differences in time to calculate the quantities on the left sides of the prognostic equations a short time *Δt* in the future

$$\frac{T(t+\Delta t,x)-T(t-\Delta t,x)}{2\Delta t} = -u_n\frac{T(t,x+\Delta x)-T(t,x-\Delta x)}{2\Delta x} + Q(x,t),$$

$$T(t+\Delta t,x) = T(t-\Delta t,x) - \frac{u_n\Delta t}{\Delta x}\left[T(t,x+\Delta x)-T(t,x-\Delta x)\right] + 2\Delta t Q(x,t).$$



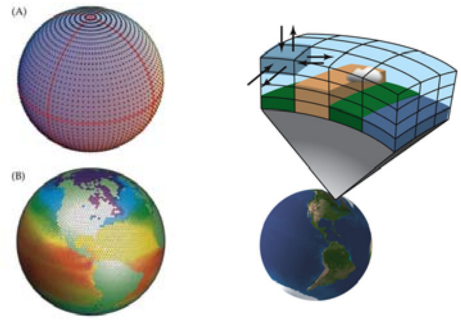

This recipe works for all of the prognostic equations
Once we have the prognostic variables at t + Δt we use the diagnostic equations to get the complete State of the model
And move on to the next time step

## Two Practical Considerations

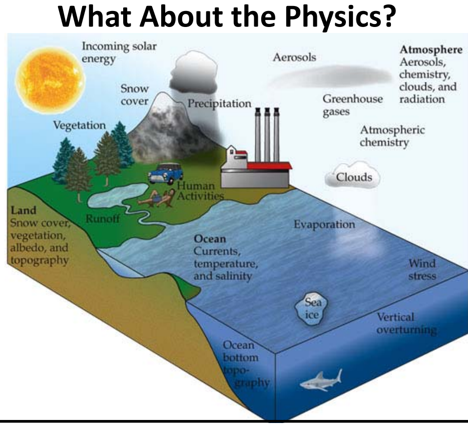
- The quantity $u\Delta t/\Delta x$ has to be less than one, or else the calculation becomes unstable and generates nonsense
- The requirement $u\Delta t/\Delta x < 1$ is called the CFL criterion.
- It means that as $\Delta x$ becomes smaller to represent more detail we need shorter (i.e., more) time steps.
- Thus if we double $\Delta x$, $\Delta y$ and $\Delta z$, the computing requirements increase not 8x but 16x, because we have to double $\Delta t$, too.
- The good news is that Moore's law states that historically computing power doubles every 18 months
- In other terms, it increases by a factor of a million in 30 years



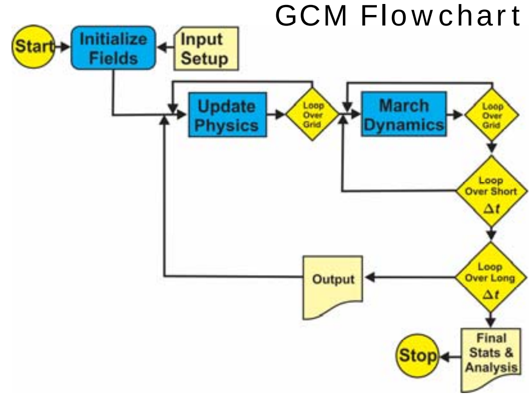
## Examples of Modern Model Grids




## What About the Physics?

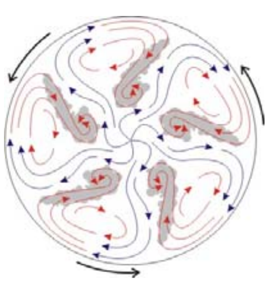



## GCM Flowchart



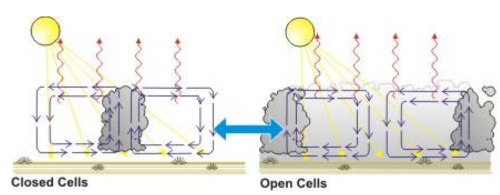

## Other Model Achitectures

- Simple "Toy" models designed to understand basic physics
- Spectral Models
  - Use sines and cosines
  - Represent waves in the atmosphere
  - The Dynamics
  - The "Physics" still use a grid
  - Fast Fourier transforms to convert between spectral and grid representations
- Finite Elements treat fluxes through surfaces of grid volumes (boxes) instead of gradients as in finite differences
- Complicated "housekeeping" limits the applicability of finite-element models



## Sensitivity to Initial Conditions

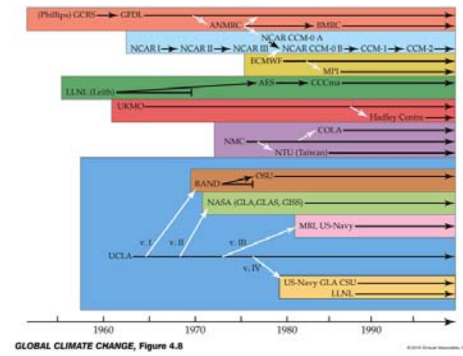
- Lorenz's (1969) simulation of convection on a primitive computer crashed
- On restart with data from before the crash, it could not reproduce earlier results
- In this case random transitions between open-cell and closed-cell convection:



## Marching and Jury Problems

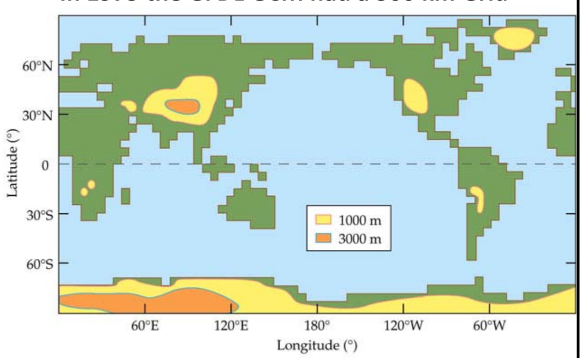
- GCM algorithms start from an ***Initial Condition***, the beginning state of the Atmosphere, and extrapolate forward in time.
  - ***Marching Problems***
- Sensitivity to Initial Conditions is what limits accuracy of day-to-day weather forecasts
- Do climate models "forget" the initial condition and become dominated by Forcing & Boundary Conditions?
  - ***Jury Problems***
- Not clear that GCM results are jury problems in the case of ocean circulations, or perhaps clouds and ice.
- Climate modelers, nonetheless, argue that they are solving jury problems

## Genealogy of GCMs




**Model creators all studied from the same texts.**

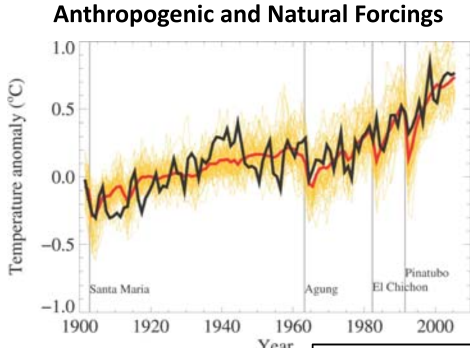
## In 1975 the GFDL GCM had a 500 km Grid




## Limitations of GCMs

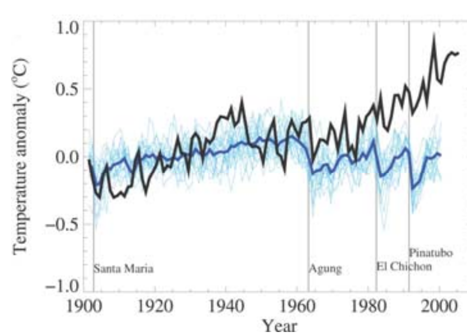
- **Don't do a particularly good job of predicting things like El Niño or the AMO.**
- **Doubts about the water vapor in the stratosphere**
- **Implicit assumption that they respond predictably to specified forcing.**
- **Parameterization of things like convection**
  - **Potentially solvable if Moore's law keeps working**

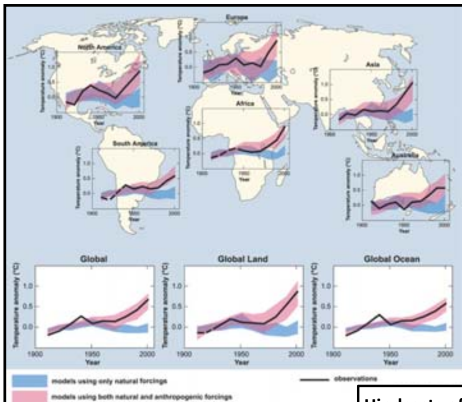
## How Good Are GCMS? Models with Anthropogenic and Natural Forcings




**Coupled Model Inter-comparison Program: CMIP**

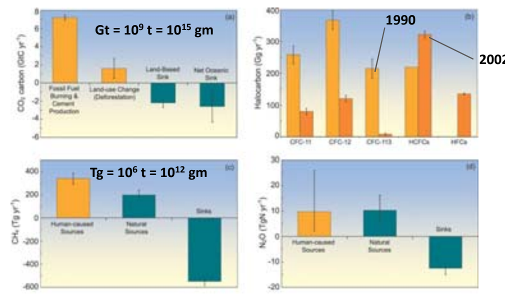
## How Good Are GCMS? Models with Natural Forcings Only





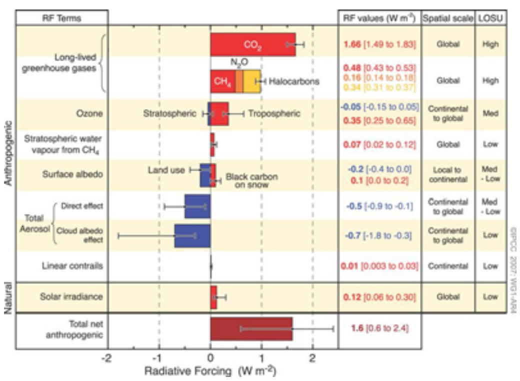

Hindcasts of 20th Century Climate by Continents

## Sources and Sinks of Green-House Gasses




Current $CO_2$ increase 3.2-4.1 Gt/yr (57% of emissions)

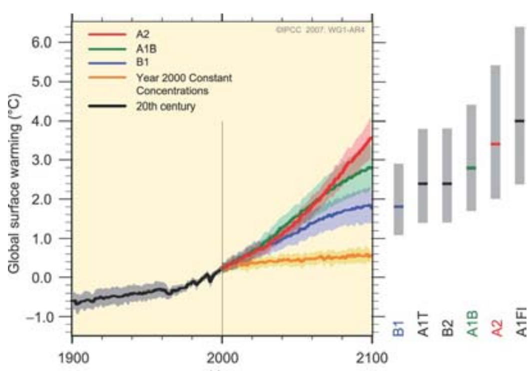
## Contributions to Radiative Forcing



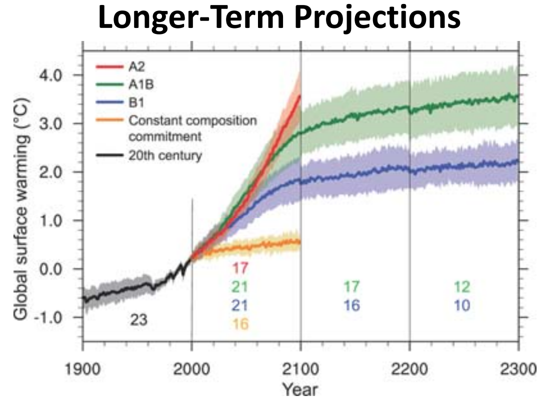

## IPCC Emission Scenarios

- **A1: Very Rapid Economic Growth Worldwide**
  - **Population peaks in mid 21st Century**
  - **Rapid introduction of new technologies**
  - **Sub-scenarios:**
    - **A1F: Fossil-fuel intensive**
    - **A1T: Alternative energy intensive**
    - **A1B: Mixed Fossil/Alternative**
- **A2: Regional development in heterogeneous world**
  - **Population grows throughout 21st Century**
- **B1: Like A1, but with less materials & energy intensive , service & information based economy.**
  - **Population peaks in mid 21st Century**
- **B2: Local solutions focusing on environmental protection and social equity**
  - **Population grows throughout 21st Century**

## The Next 100 Years by Emission Scenario




## Longer-Term Projections



## Summary

- Models used to understand & predict
  - Partial derivatives and partial differential equations
- Grid-point GCMs
  - Prognostic and diagnostic equations
  - Leap-frog time marching
  - Dynamics: Wind and Pressure
  - Physics: Radiation, Convection, Surface Transports
  - Role of resolution: CFL Criterion & Moore's Law
- Toy, spectral, and finite element models
- Marching & Jury Problems.
  - Sensitivity to Initial Conditions
- Models reproduce 20th Century climate, but only with anthropogenic forcing
- Greenhouse gasses. $CO_2$, $NO_2$, $CH_4$, CFCs
  - $CO_2$ most important: Increasing 1.5 ppm/yr or 3-4Gt/yr
- Emission Scenarios:
  - A1F, A1B, A1T: Globalization with fossil fuels, mixed and green energy. Population maximum in mid 21st Century
  - A2: Slower and more fragmented economic growth. Continuing population growth
  - B1: Greener globalization, same population as A1
  - B2: Lower-tech with growing population through 2100
- Predicted warming ranges from 2C (B1) to 4C (A1F)