The mid-term exam will be 40-50% multiple choice and the remainder (60-50%) longer (1-2 paragraphs) answer, problems, and mathematical derivations.

Things to know:

1. As a review of background material you need to:
   a. Understand the SI units and prefixes used to measure atmospheric quantities
   b. Be able to sketch the vertical structure of the middle-latitude (i.e. standard atmosphere) troposphere and lower stratosphere including values of surface temperature, lapse rate and tropopause height
   c. Know generally the scales of motion that characterize middle-latitude, synoptic-scale weather systems.
   d. Be able to explain in general terms the roles the tropics, middle latitudes and the poles in the Atmosphere's general circulation
   e. Be able to sketch the "Big Picture with the Atmosphere".
2. Primitive equations: Atmospheric Navier-Stokes Equations with explicit velocity accelerations and no filtering, no use of the geostrophic approximation, and no other kinds of balanced flow approximations. For example:
   a. Netwon's second law: $\rho \mathrm{D}\vec{v} / \mathrm{D}t = \vec{F}$ = sum of imposed forces
   b. Gas law for dry air: $p\alpha = p / \rho = R_d T$
   c. First law of Thermodynamics: $c_v \mathrm{D}T / \mathrm{D}t + p\mathrm{D}\alpha / \mathrm{D}t = J$
   d. Mass continuity $\mathrm{D}(\rho\vec{v}) / \mathrm{D}t = 0$
3. Be able to describe Cartesian coordinates as used in dynamic meteorology. Know the meanings of the adjectives ***Zonal***, ***Meridional***, and ***Vertical*** in this context, as well as the conventional notation for the corresponding distances (x, y, z) and velocity components (u, v, w).
4. Know that the meteorologically significant forces are and be able to explain how they are incorporated into the Navier-Stokes equations:
   a. Gravity, $-g\hat{k}$
   b. Pressure gradient, $-\rho^{-1}\nabla p$
   c. Coriolis, $-2\Omega\sin\varphi\hat{k}\times\vec{v}_2 = -f\hat{k}\times\vec{v}_2$
   d. Understand why only the locally vertical component of the Earth's rotation produces a meteorologically significant Coriolis force.
   e. Know how friction can be represented using a drag coefficient, bulk aerodynamic friction, or eddy diffusion.
5. Be able to describe the action of the Coriolis force in an idealized system of frictionless, moving objects on a rotating turntable.

6. Understand the mathematics of uniform circular motion at a fixed radius. Be able to differentiate once to get the velocity of the rotating object and twice to get the acceleration. Be able to describe both the magnitudes and directions of these quantities.
7. Be able to derive the Coriolis force on a rotating Earth in terms of the rate of change of a vector in the rotating and fixed reference frames, using $(d\mathbf{R}/dt)_{\mathrm{FIX}} = (d\mathbf{R}/dt)_{\mathrm{ROT}} + \mathbf{\Omega}\times\mathbf{R}$.
8. Hydrostatic law: The pressure is simply the weight of the fluid above a give point. $\partial p/\partial z = -g\rho$ in height coordinates, or $\partial\phi/\partial p = -\alpha = -RT/p$ in pressure coordinates.
9. Be able to integrate the hydrostatic law in an isothermal atmosphere to get the pressure as a function of height.
10. Scale height, *H*: The altitude interval over which the pressure (or density) decreases by a factor of $e^{-1} = 0.3679$ in an isothermal atmosphere. $H = RT/g$.
11. Summary of differentiation

- Derivatives represent rates of change of functions.
- Easily remembered recipes for calculating them from functions:
- $d\,x^n/dx = nx^{n-1}$
- d(cos x)/dx = -sinx; d(sin x)/dx = cos x
- $d\,e^x/dx = e^x$; d(ln x)/dx = 1/x
- Chain rule
- Rules for sums, quotient, and products

12. Summary of integration

- Integration is the reverse of differentiation
- Areas under curves
- Constants of integration determined by limits
- Standard formulas
- Euler's formula $e^{ix} = \cos x + i \sin x$
- Used to solve partial differential equations for wave motion

13. Summary of vectors

- Vector: Magnitude and direction
- Scalar: Magnitude only
- Project onto components along coordinate axes
- Vector wind = meteorological wind + 180^o
- Sums and differences: Add and subtract components
- Derivative; Derivative of components
- Inner (dot) product: Multiply parallel components (Scalar)
- Outer (cross) product: Differences of products of perpendicular components (Vector)
- Divergence and curl, like inner and outer products but using derivatives
- Divergence: expansion and contraction
- Curl: Rotation and some deformation
- Gradient points uphill in the coordinate directions for functions of spatial coordinates
- Laplacian is the divergence of the gradient

14. Know the difference between Lagrangian (individual) and Eulerian (local) derivatives, $\mathrm{D}()/\mathrm{D}t = \partial()/\partial t + u\partial()/\partial x + v\partial()/\partial y + w\partial()/\partial z$. Advection is the part of the Lagrangian derivative that stems from the motion of the air parcel, $= u\partial(\ )/\partial x + v\partial(\ )/\partial y + w\partial(\ )/\partial z$. The Eulerian, or local, derivative is the time rate of change at a fixed location, $\partial()/\partial t$.
15. Understand the differences among ***Diagnostic*** equation, that contain no time derivatives, ***Prognostic*** equations that do contain time derivatives. Prognostic equations in **Initial-Value Form (IVF)** that have the first-order time derivative of a single variable on the left side of the equal sign and spatial derivatives and values of variables on the right side, but no time derivatives.
16. Understand how the universal gas law can be specialized to the gas laws for dry air and water vapor. Know generally how virtual temperature of moist air works and its effect on density.
17. Be able to interpret the terms in the First Law of thermodynamics when it is expressed as changes in temperature and specific volume, i.e. changes in internal energy at constant volume, pressure work, and heating.
18. Know the difference between ***Diabatic*** and ***Adiabatic*** processes. ***Isotherma***l and ***Isobaric*** processes take place at constant temperature and pressure, respectively. An ***Isentropic*** process occurs with no heat added or removed; that is it is adiabatic.
19. Using the gas law, be able to transform the 1st Law into a more meteorologically useful from: $c_p \mathrm{D}T/\mathrm{D}t - \alpha \mathrm{D}p/\mathrm{D}t = J$, where $c_p = c_v + R$
20. Understand how to derive the dry adiabatic lapse rate from the 1st Law in a hydrostatic atmosphere. Know the relationship between the actual lapse rate and the dry adiabatic lapse rate in stable, neutral and unstable stratification.
21. Know how potential temperature conservation is derived from the 1st Law.
22. Understand the use of pressure as a vertical coordinate changes the pressure gradient to $(1/\rho)\partial p/\partial x = g\partial z_p/\partial x = \partial\phi/\partial x$ and mass continuity to $\partial u/\partial x + \partial v/\partial y + \partial\omega/\partial p = 0$. It removes explicit dependence on density from the momentum and mass continuity equations. The "vertical" velocity in pressure coordinates is "omega", $\omega \equiv \mathrm{D}p/\mathrm{D}t \approx -g\rho w$. Use of pressure coordinates also removes time dependence from the mass continuity equation.
23. In summary, the advantages of pressure coordinates are:
    a. No density in the momentum equations
    b. Thermodynamic energy equation becomes an IVF prognostic equation (ω replaces Dp/Dt) and can be written in terms of $\partial\phi/\partial p$ rather than temperature for hydrostatic motions.
    c. Mass continuity equation becomes diagnostic with no explicit density or specific volume dependence.
24. Know how the ***Geostrophic Approximation*** derives from the momentum equations. The geostrophic wind blows perpendicular to the pressure gradient such that the Coriolis force balances the pressure gradient: $u_g = -f^{-1}\,\partial\phi/\partial y, v_g = f^{-1}\,\partial\phi/\partial x$. A necessary condition for geostrophic balance is small Rossby Number = $V/fL \ll 1$.

25. Be able to compute the geostrophic wind given the height gradient in pressure coordinates or the pressure gradient and density in height coordinates. Know that 1 hPa = 1 mb = 10^2 Nt m^{-2} = 10^2 kg m s^{-1} m^{-2}.
26. Understand the **Cyclostrophic Approximation** in which the wind blows in such a small circle that the Coriolis term is negligible, $v^2/r = \partial\phi/\partial r$. A necessary condition for cyclostrophic balance is large Rossby Number. The cyclostophic wind may blow cyclonically or anticylonically, but it can be present only in low pressure systems.
27. The ***Inertia Circle*** or inertia oscillation occurs when air moves in an anticyclonic circle (always) with frequency *f*, such that $fv = v^2/r$.
28. **Gradient Balance:** Wind such that both the centripetal acceleration and the Coriolis acceleration are important: $v^2/r + fv = \partial\phi/\partial r$. Gradient balance prevails when the Rossby number is approximately one. Be able to relate the gradient and geostrophic winds with the cute derivation that we did in class $v_G = v_C(v_C / f_0 r) / \frac{1}{2}\left(1 \pm \sqrt{1 + 4v_C^2 / f_0^2 r^2}\right)$, or $v_G = v_g / \frac{1}{2}\left(1 \pm \sqrt{1 + 4v_g / f_0 r}\right)$ and be able to explain how for large Rossby number the flow is cyclostrophic and for small Rossby number it is geostrophic. You will need the identity $fv_g = \partial\phi/\partial r = v_G^2 / r$. Recall that the geostrophic Rossby number is $v_g / f_0 r$ and the cyclostrophic Rossby number is $v_C / f_0 r$. Gradient winds <u>normally</u> blow cyclonically around lows and anticyclonically around highs, as do geostrophic winds.
29. For <u>normal gradient winds</u> with a fixed magnitude of the pressure gradient, cyclonic winds are weaker than the geostrophic wind and anticyclonic winds are stronger. This is true, because both the Coriolis and centripetal accelerations balance the pressure gradient in the former; whereas the only the Coriolis acceleration balances the combined pressure gradient force and centripetal acceleration in the latter.
30. Know that <u>anomalous gradient winds</u> result from choice of the negative root in the quadratic equation for the gradient wind. The only physically possible example gives rise to anticyclonic flow around a low that approaches an inertial oscillation far from the center. In the limiting case of large Rossby number, it is analogous to anticyclonic cyclostrophic wind blowing around a low. In cyclonic anomalous flow around a high, all forces would be directed outwards, but it is possible to have anticyclonic anomalous flow around a high at low Rossby number. This case also approximates the inertia oscillation rather than the geostrophic wind.
31. Be able to sketch the balances of forces for circular motion around low- and high-pressure centers in both northern and southern hemispheres.
32. Given the gradient wind equation, be able to show that the limiting magnitude of the Rossby number is ¼ for anticyclonic flow around a high pressure system.
33. Understand how the hypsometric equation $\phi_2 - \phi_1 = R\overline{T}_{1,2}\ln(p_1/p_2)$ derives from the hydrostatic equation.
34. Be able to derive the thermal wind equation, $v_2 - v_1 = Rf^{-1}(\partial\overline{T}_{1,2}/\partial x)\ln(p_1/p_2)$, and similarly for $u_2 - u_1$, from the hypsometric equation. Thermal wind is the vertical shear of the horizontal wind due to horizontal gradients of temperature. It bears the same qualitative relationship to the thickness (temperature) field as the geostrophic wind does to the height (or geopotential) field.

35. Be able to explain why the thermal ***Backs*** [turns anticlockwise (cyclonically) with height] in cold advection and ***Veers*** [turns clockwise (anticyclonically) with height] in warm advection in the Northern Hemisphere.

36. Divergence: $\nabla_H \vec{v} = \partial u / \partial z + \partial v / \partial y$. A flow with zero divergence is said to be ***Nondivergent***, and a flow with nonzero divergence is said to ***Divergent***.

37. Relative vorticity: $\nabla_H \times \vec{v} = (\partial v / \partial x - \partial u / \partial y)\hat{k}$. A flow with zero vorticity is said to be ***Irrotational***, and a flow with nonzero vorticity is said to be ***Rotational***.

38. Be able to show that wind derived from a streamfunction $u = -\partial \psi / \partial y, v = \partial \psi / \partial x$ is always nondivergent and that wind derived from a velocity potential $u = -\partial \chi / \partial x, v = -\partial \chi / \partial y$ is always irrotational.

39. Know what Helmholtz's Theorem is and why it is important. Also, be able to sketch divergent, rotational and deformational flows and know the difference between streamlines and trajectories.