The exam will be 30% multiple choice and the remainder (70%) longer (1-2 paragraphs) answer, problems, and mathematical derivations.

Things to know:

1. Understand how to derive the vorticity equation in pressure coordinates.
2. Understand the derivation of the continuity equation in pressure coordinates and how it simplifies the result (i. e., changes it to pure pressure-coordinate divergence) relative to the height-coordinate equivalent (which contains the individual derivative of density, or at least vertical advection of density.
3. Understand how we transform the momentum equations to absorb the advective and geopotential terms into gradients of total energy and plus the cross velocities times the absolute vorticity. For example: $\partial u / \partial t + \zeta v - \partial[\frac{1}{2}(u^2 + v^2) + \phi] / \partial x$.
4. In the vorticity equation, be able to identify the Lagrangian derivative, vertical advection, vorticity stretching, and tilting terms and know why and how each is important.
5. Relative vorticity: $\zeta = \partial v / \partial x - \partial u / \partial y$. A flow with zero vorticity is said to be "irrotational," and a flow with nonzero vorticity is said to be "rotational."
6. Planetary vorticity: The component of the rotation of the Earth around the local vertical at latitude φ. It is numerically equal to the Coriolis parameter $f = 2\Omega \sin\varphi$, where Ω is the angular velocity of the Earth, 2π radians in 86400 s = 24 h.
7. Absolute vorticity: The sum of relative and planetary vorticity $\eta = \zeta + f$.
8. Circulation: Given a closed path, the circulation is the integral around the path of the velocity component locally parallel with the path times an element of length along the path. $C = \oint_P \vec{v} \cdot d\vec{\ell}$.
9. Circulation theorem: The circulation around a closed path fixed to the Earth is equal to the total relative vorticity enclosed by the path, i.e. equal to the integral of the vorticity over the area enclosed, $\oint_P \vec{v} \cdot d\vec{\ell} = \iint_A \zeta da$, or the average vorticity enclosed times the area enclosed.
10. Be able to use the circulation theorem to derive the flow in a Rankine vortex, where $\zeta = \zeta_0$ for $r \leq A$ and $\zeta = 0$ for $r > A$. Know that the "free-vortex" flow in the outer region in irrotational.
11. Conserved Quantity: A quantity whose value does not change following an air parcel (i.e. D( )/D*t* =0), under specified conditions, even though the value may change if those conditions are violated. For example, absolute vorticity is conserved in nondivergent flow. Examples:
    a. Potential temperature: $\theta = T(p_0/p)^{R/c_p}$ is conserved in the absence of conductive or radiative heating.
    b. Potential vorticity: $\Pi = (\zeta + f)/\Delta p$ or $\Pi = (\zeta + f)/h$ because $\Delta p = -g\rho h$ is conserved in the absence of rotational friction or mass sources and sinks that change Δp or h.
12. Quasi-geostrophic (QG) Approximation: An elaboration of geostrophic balance in which the meridional extent of the disturbances is much less than the Earth's radius. Under this

approximation, acceleration of the geostrophic wind is due to the Coriolis force caused by the ageostrophic wind; only the ageostophic wind is divergent; and the meridional gradient of planetary vorticity (β) is negligible except where it is advected by the meridional quaigeostrophic wind. The QG momentum equation is $\mathrm{D}\vec{v}_q/\mathrm{D}t + f\hat{k}\times(\vec{v}-\vec{v}_q)+\beta y\hat{k}\times\vec{v}_q = 0$. Here, again, the acceleration of the QG wind is proportional to the Coriolis force that arises from the ageostrophic wind, $f\mathbf{k} \times \mathbf{v}_a$. Be able to apply this idea to streamwise acceleration or streamwise deceleration and cyclonic and anticyclonic curvature of the QG wind.

13. The ageostrophic mass continuity equation is $\nabla\bullet\mathbf{v} = \nabla\bullet(\mathbf{v}_q+\mathbf{v}_a) = \nabla\bullet\mathbf{v}_a = -\partial\omega/\partial p$ and since $\{\mathbf{v}_a\} \sim$ Ro $\{\mathbf{v}_q\}$ the pressure-coordinate vertical velocity is much weaker than we would think from naïve scaling.
14. In the QG individual derivative we neglect advection by the ageotrophic wind because the $\mathbf{v}_a$ scales as Ro times $\mathbf{v}$ or $\mathbf{v}_q$. $D_q(\,) = \partial(\,)/\partial t + u_q\partial(\,)/\partial x + v_q\partial(\,)/\partial y$. Similarly we generally neglect vertical advection because ω scales as Ro(H/L) because all of the QG divergence is thrown into the ageostrophic part of the wind. Note that H/L is called the "aspect ratio."
15. Essential features of the QG suite of approximations are, near geostrophic balance computed using constant $f_0 = 10^{-4}$. The beta plane approximation $f = f_0 + \beta y$ is used elsewhere, as in the QG vorticity equation. Other essential aspects are hydrostatic balance; use of pressure coordinates; and (generally) adiabatic thermodynamics (see item 31).
16. QG vorticity equation: $\mathrm{D}_q\zeta_q/\mathrm{D}t - f_0(\partial\omega/\partial p) + \beta v_q = 0$. Note that it is the (constant) mean Coriolis parameter that multiplies the divergence, not the total meridionally varying Coriolis parameter. Be able to derive the QG vorticity equation from the QG momentum equation expressed in component form.
17. Beta-plane approximation: Representation of the Coriolis parameter using a truncated Taylor series centered at a mean latitude, such that $f = f_0 + \beta y$. Know why $\beta y << f_0$.
18. Understand the derivation of the Isallobaric wind and know why it converges into deepening low pressure centers and diverges from building high pressure centers.
19. Understand, and be able to demonstrate, why a geostrophic (not quasigeotrophic, which by definition is nondivergent) wind from the south is convergent and a geostrophic wind from the north is divergent. Be able to relate this argument to weather encountered as the wind veers from south to north. This phenomenon appears in the QG context this divergence projects onto the ageostrophic wind such that the ageostrophic wind is convergent when the QG wind is poleward and divergent when it is equatorward,
20. Know how combining the thermodynamic energy and pressure coordinate hydrostatic equations produces $\mathrm{D}(\partial\phi/\partial p)/\mathrm{D}t + \sigma\omega = -\kappa J/p$.
21. Know that the quasi-geostrophic wind is nondivergent because we use constant f to compute it, so that all of the divergent flow is in the ageostrophic part of the wind.
22. Be able to derive the geostrophic divergence on a beta plane.
23. Be able to use the idea that the acceleration of the geostrophic wind is proportional to the Coriolis force from the ageostrophic wind to deduce the sense of the ageostrophic wind for streamwise accelerating, streamwise decelerating, or curved (cyclonic or anticylonic) flow. Also

be able to deduce the locations of rising and sinking motion for accelerated flows just below the tropopause.

24. Since the ageostrophic wind is a factor of Ro (=*V/fL*) smaller than the geostrophic wind and the aspect ratio = *H/L* of large scale flows is small, mass continuity implies that omega is so small that we can neglect vertical advection <u>except</u> in the thermodynamic equation.
25. Barotropic flow has no horizontal temperature gradients. Temperature is a function only of the vertical coordinate (height or pressure) and the thermal wind is zero.
26. Baroclinic flow has a horizontal temperature gradient such that the thermal wind is nonzero.
27. The full QG equations for atmospheric motion are:
    a. Momentum: $\mathrm{D}_q\vec{v}_q/\mathrm{D}t + f\hat{k}\times(\vec{v}-\vec{v}_q)+\beta y\hat{k}\times\vec{v}_q = \mathrm{D}_q\vec{v}_q/\mathrm{D}t + f\hat{k}\times\vec{v}_a+\beta y\hat{k}\times\vec{v}_q = 0$
    b. Vorticity: $\mathrm{D}_q\zeta_q/\mathrm{D}t - f_0(\partial\omega/\partial p)+\beta v_q = 0$
    c. Mass continuity: $\partial u_a/\partial x + \partial v_a/\partial y + \partial\omega/\partial p = 0$
    d. Thermodynamic energy: $\mathrm{D}_q(\partial\phi/\partial p)/\mathrm{D}t + \sigma\omega = -\kappa J/p$
28. Know how to eliminate ω between the QG vorticity and thermodynamic energy equations to produce the QG geopotential tendency equation:

$$\frac{1}{f_0}\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\frac{\partial\phi}{\partial t} =$$
$$-\vec{v}_q\cdot\nabla\left[\frac{1}{f_0}\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+f_0+\beta y\right]-\frac{\partial}{\partial p}\left[\vec{v}_q\bullet\nabla\left(\frac{f_0}{\sigma}\frac{\partial\phi}{\partial p}\right)\right]$$

29. Know that the right side of the QG geopotential tendency equation can then be manipulated using the thermal wind equation to produce the QG potential vorticity equation:

$$\frac{\partial}{\partial t}\frac{1}{f_0}\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{f_0^2}{\sigma}\frac{\partial^2\phi}{\partial p^2}\right)+\vec{v}_q\cdot\nabla\left(\frac{1}{f_0}\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{f_0^2}{\sigma}\frac{\partial^2\phi}{\partial p^2}\right)+f_0+\beta y\right)=0$$

30. Know how elimination of $\partial\phi/\partial t$ between the vorticity and thermodynamic energy equations yields the QG omega equation:

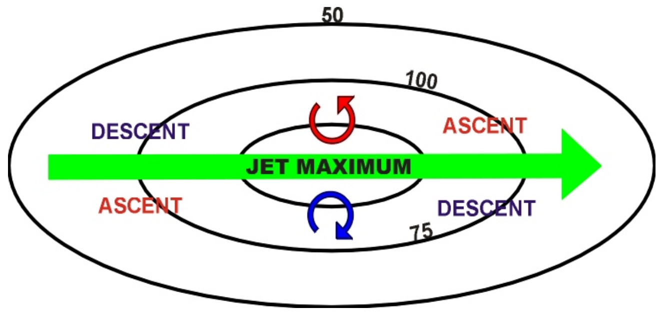
$$\left[\frac{1}{f_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\right]\omega = \frac{f_0}{\sigma}\frac{\partial}{\partial p}\left[\vec{v}_q\cdot\nabla\left(\frac{1}{f_0}\nabla^2\phi+f_0+\beta y\right)\right]$$
$$+\nabla^2\left[\vec{v}_q\cdot\nabla\left(-\frac{\partial\phi}{\partial p}\right)\right]-\kappa\nabla^2\left(\frac{J}{p}\right)$$

31. Application of the chain rule to the right side of 30 and neglect of some (sort-of) small terms simplifies the QG ω-equation to:

$$\left[\frac{1}{f_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\right]\omega = \frac{f_0}{\sigma}\left[\frac{\partial\vec{v}_g}{\partial p}\cdot\nabla\left(\frac{1}{f_0}\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right)+f_0+\beta y\right)\right]$$

Thus, when the horizontal wind is increasing upward and blowing from anticyclonic to cyclonic vorticity the forcing on the right is negative. Since the left sides of 28-31 are Poisson equations, the solutions are proportional the negative of the forcing, and $\omega > 0$ will be positive, implying descent. Conversely if the wind blows from cyclonic to anticyclonic the forcing will be positive, such that $\omega < 0$ implies ascent.

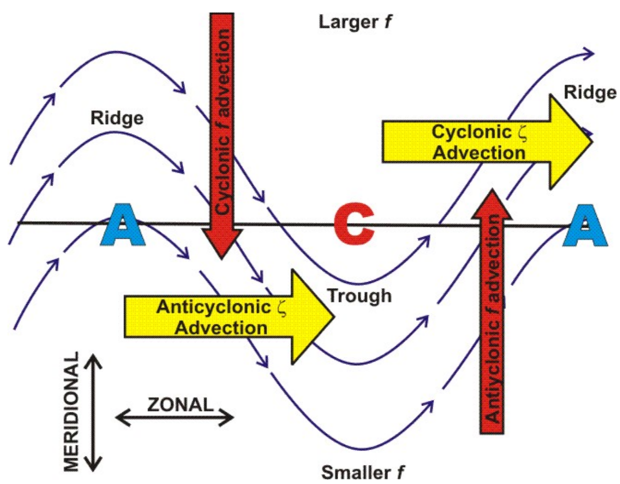
32. You should be able to use either the ω Equation or the ageostrophic wind to explain the four-quadrant model of a straight Jet-Stream wind maximum (or Jet Streak), recalling that the Jet Streak itself moves more slowly than the wind at the level of the strongest wind.



33. Know and understand the middle-latitude approximations that underlie the quasigeotrophic system of approximations:

- Hydrostatic
- Wind is predominantly geostrophic
- Use constant $f = f_0$ in computing wind and vorticity from geopotential
- Adiabatic motions (almost always)
- Beta Plane $f \approx f_0 + \beta y$
- Small (order Ro relative to the QG wind) divergent (ageostrophic) component resulting in small vertical advection everywhere except in the thermodynamic energy equation.

34. Be able to explain how zonal advection of relative vorticity by the (mean) zonal flow and meridional advection of planetary vorticity by the (perturbation) meridional flow combine to cause a sinusoidal pattern of streamlines to move more slowly than a westerly mean flow and may even cancel completely to keep the waves stationary.



NOTE: Items 35-41 will not be covered on Exam #2, but will be covered on the FINAL

35. Understand and be able to explain the differences among the various forms of energy in the Atmosphere: **Potential**, **Kinetic,** and **Thermodynamic** (**Enthalpy**) and know the mathematical expressions for each.

36. Know that (specific) energy in J kg^{-1} is dimensionally equivalent to velocity squared ($m^2\ s^{-2}$) and that (specific) power in W kg^{-1} is dimensionally velocity squared per second ($m^2\ s^{-3}$).

37. Be able to derive and explain the quasigeostrophic energy equation from the QG momentum and mass continuity equations.

38. Know why the geopotential work term on the right side of the QG energy equation contains the ageostrophic wind, $\mathbf{v}_a$, rather than the quasigeostrophic wind, $\mathbf{v}_q$.

39. Know the difference between advective and flux forms of the individual (Lagrangian) derivatives and how to use the continuity equation to go from one form to the other.

40. Understand how to derive the atmospheric energy equation in pressure coordinates without making the QG approximation, including use of the continuity equation to get a three dimensional geopotential work term plus a pressure work term $-\alpha\omega = -\omega RT/p$, that cancels a corresponding term in the Thermodynamic Energy Equation.
41. Given the Kinetic Energy Equation that results from the foregoing analysis and the Thermodynamic Energy Equation, be able to eliminate the pressure work term to get a Total Atmospheric Energy Equation in which the diabatic heating is the only forcing. Note that in class we omitted friction.

You should be able to:

1. Work the problems from the homework on the exam, or similar ones.
2. Be able to calculate circulation and/or area average vorticity given a closed path and either the area average vorticity or the circulation.
3. Be able to calculate simple wind distributions from the vorticity using the circulation theorem.
4. Be able to calculate geostrophic wind from the geopotential in pressure coordinates. Remember, no factor of 100 to convert millibars, and 1000 km = 10^6 m.
5. Be able to derive a vorticity equation from the momentum equations in height coordinates.
6. Be able to explain in words how the geopotential tendency, quasi-geostrophic potential vorticity equation and omega equation can be derived from the quasi-geostrophic vorticity and thermodynamic energy equations.
7. Be able to explain, in terms of barotopic, nondivergent vorticity advection, why a sinusoidal geopotential field can move more slowly than the mean zonal <u>westerly</u> wind, even to the extent of remaining stationary or retrograding upwind.
8. Be able to explain the pattern of ascent ($\omega < 0$) and descent ($\omega > 0$) in jet-stream flow using the QG ω Equation or the QG momentum equation and ageotrophic winds.