

FINAL EXAM Monday, 11DEC17, 12:00 Noon to 2:00 PM, AHC5-357. The exam will consist of 30 multiple choice questions, worth 2 points each, and 10 problems, short answer, or draw and explain, worth 15 points each. Of the latter set of questions you must choose 6 to answer. In all, the exam will be worth 150 points, or about 33% of your grade in the course. **The final is cumulative.** It may include edited problems/questions from the homework and the two in-class exams.

Specific topics since the last exam that you should know:

1. Understand and be able to explain the differences among the various forms of energy in the Atmosphere: **Potential, Kinetic, and Thermodynamic (Enthalpy)** and know the mathematical expressions for each.
 2. Know that (specific) energy in J kg^{-1} is dimensionally equivalent to velocity squared ($\text{m}^2 \text{s}^{-2}$) and that (specific) power in W kg^{-1} is dimensionally velocity squared per second ($\text{m}^2 \text{s}^{-3}$).
 3. Be able to derive and explain the quasigeostrophic energy equation from the QG momentum and mass continuity equations.
 4. Know why the geopotential work term on the right side of the QG energy equation contains the ageostrophic wind, \mathbf{v}_a , rather than the quasigeostrophic wind, \mathbf{v}_q .
 5. Know the difference between advective and flux forms of the individual (Lagrangian) derivatives and how to use the continuity equation to go from one form to the other.
 6. Understand how to derive the atmospheric energy equation in pressure coordinates without making the QG approximation, including use of the continuity equation to get a three dimensional geopotential work term plus a pressure work term $-\alpha\omega = -\omega RT/p$, that cancels a corresponding term in the Thermodynamic Energy Equation.
 7. Given the Kinetic Energy Equation that results from the foregoing analysis and the Thermodynamic Energy Equation, be able to eliminate the pressure work term to get a Total Atmospheric Energy Equation in which the diabatic heating is the only forcing. Note that in class we omitted friction.
1. Understand and be able to explain derivation of the shallow-water, barotropic model from the more general Navier-Stokes equations as applied to the Atmosphere.

Derivation of QG Energy Equation (Reminder):

Start with the QG momentum equations in component form:

$$\frac{\partial u_q}{\partial t} + u_q \frac{\partial u_q}{\partial x} + v_q \frac{\partial u_q}{\partial y} - f v_a - \beta y v_q = 0,$$

$$\frac{\partial v_q}{\partial t} + u_q \frac{\partial v_q}{\partial x} + v_q \frac{\partial v_q}{\partial y} + f u_a + \beta y u_q = 0.$$

Multiply the zonal momentum equation by u_q and the meridional momentum equation by v_q :

$$\frac{\partial u_a^2}{\partial t} + u_a \frac{\partial u_a^2}{\partial x} + v_a \frac{\partial u_a^2}{\partial y} - f_0 u_a v_a - \beta y u_a v_a = 0,$$

$$\frac{\partial v_a^2}{\partial t} + u_a \frac{\partial v_a^2}{\partial x} + v_a \frac{\partial v_a^2}{\partial y} + f_0 u_a v_a + \beta y u_a v_a = 0.$$

Summing:

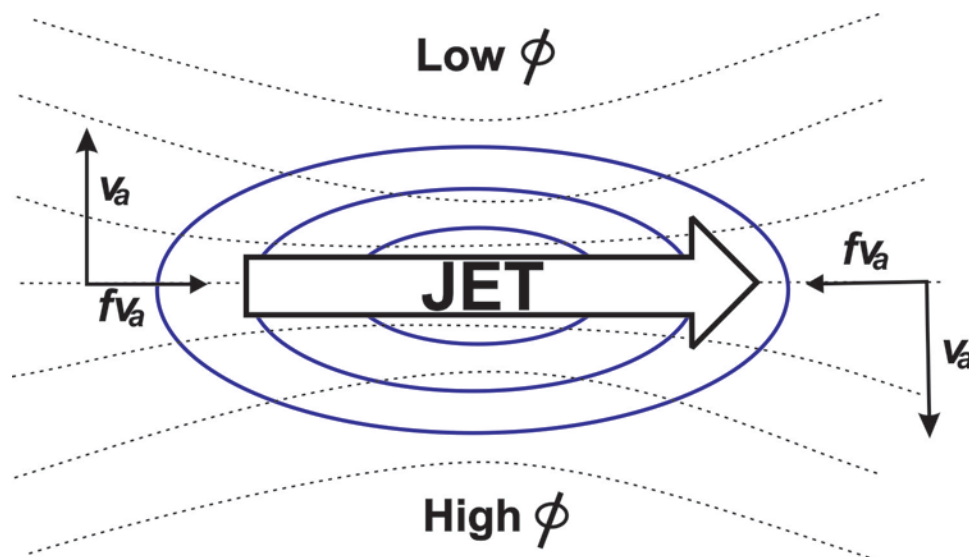
$$\frac{\partial}{\partial t} \frac{1}{2} (u_a^2 + v_a^2) + u_a \frac{\partial}{\partial x} \frac{1}{2} (u_a^2 + v_a^2) + v_a \frac{\partial}{\partial y} \frac{1}{2} (u_a^2 + v_a^2) - f_0 u_a v_a + f_0 v_a u_a = 0,$$

or,

$$\frac{\partial K_a}{\partial t} + u_a \frac{\partial K_a}{\partial x} + v_a \frac{\partial K_a}{\partial y} + f_0 [-u_a v_a + u_a v_a] = 0,$$

$$\frac{\partial K_a}{\partial t} + u_a \frac{\partial K_a}{\partial x} + v_a \frac{\partial K_a}{\partial y} + v_a \frac{\partial \phi}{\partial y} + u_a \frac{\partial \phi}{\partial x} = 0,$$

where $K_a = \frac{1}{2}(u_a^2 + v_a^2)$ is the QG kinetic energy. Thus, (downhill) ageostrophic flow from high geopotential to low geopotential increases K_a ; whereas (uphill) ageostrophic flow from low geopotential



to high decreases K_a , as the now familiar depiction of a westerly jet streak illustrates:

In addition to understanding how flow through a jet streak works you should also be able to visualize ageostrophic wind and vertical velocity when the QG wind blows through troughs and ridges. The key is that the Coriolis force points toward the center of rotation so that it bends the flow around the curve in both cases. Thus it points downstream in a ridge and upstream in a trough, causing the actual wind to be stronger than the QG wind in ridges and weaker in troughs, consistent with the gradient wind. As the wind accelerates entering a ridge and decelerates leaving the ridge, it produces upper divergence,

leading to mid tropospheric ascent upstream and descent downstream. Conversely in a trough the wind decelerates as it enters and accelerates as it leaves, causing descent upstream and ascent downstream.

