

Work these problems neatly on separate sheets of paper. Be sure to put your name in the upper right corner of each sheet and to number them. This homework is due a week from Monday. You may work together, but don't copy. I'm available for advice.

1. Consider a situation where the hand of Wotan reaches down from Asgard and abruptly switches on a uniform pressure gradient such that  $\rho_0^{-1} \partial p / \partial y = -f_0 U_g$ . At that first moment, the winds are identically zero. The governing equations for the horizontal wind are:

$$\begin{aligned} \frac{du}{dt} - f_0 v &= 0, \\ \frac{dv}{dt} + f_0 u &= f_0 U_g. \end{aligned}$$

Here  $u$  and  $v$  are the velocity components, and  $f_0$  is the constant Coriolis parameter. In this problem we will treat the derivatives as ordinary total derivatives neglecting advection. Obtain expressions for  $u(t)$  and  $v(t)$ . Briefly describe and interpret the result.

Inhomogeneous systems like this have a particular solution that produces the forcing when substituted into the equations and a complementary solution that satisfies the homogeneous equations. The particular solution is a homogeneous solution of the "annihilator," a combination of derivatives such that yields zero when it is applied to the right-hand side. In this case the annihilator is  $d(\ )/dt$  so that the particular solution is some constant, or other. The complete solution is a superposition of the two that satisfies the initial conditions,  $u(0) = v(0) = 0$ .

2. Revisit the above problem, but with a sinusoidal forcing such that the governing equations are:

$$\begin{aligned} \frac{du}{dt} - f_0 v &= 0, \\ \frac{dv}{dt} + f_0 u &= f_0 U e^{i\omega t}. \end{aligned}$$

Here the annihilator is  $d^2(\ )/dt^2 + \omega^2(\ ) = 0$ , such that the particular solution has the form  $Ae^{i\omega t}$ , where  $A$  is a complex amplitude. Substitute this expression into the governing equations and find the particular solution (i.e.,  $A$  as a function of  $\omega$ ,  $f_0$ , and  $U$  times  $e^{i\omega t}$ ). For extra credit, combine your result with the complementary solution to obtain a complete solution that satisfies the boundary condition  $u(0) = v(0) = 0$ . Describe your result in words.