Syllabus, Advanced Calculus, MAA 4211, Section U1, Class Number 11850

Book: “Mathematical Analysis: an introduction” by Andrew Browder, ISBN 0-387-94614-4.

Description.

The course on Advanced Calculus is devoted to rigorous presentation of the basics of Mathematical Analysis of real valued functions of one (real) variable from the standpoint of contemporary/modern Mathematics. It comes as a natural continuation of the Introduction to Advanced Mathematics course taught in Fall'06, and relies on the previous experience of the students from Calculus I and II (in the form accepted at FIU). The Advanced Calculus students are expected to be skillful in the computational aspects of Calculus I and II, and to be aware of the basic theorems in those courses. The purpose of Advanced Calculus will be to give proofs of those (and even more) theorems, and to put them in a right perspective (the theorems in question form a body of a scientific knowledge rather than a list of recipes for doing exercises and passing exams!)

The topics to be covered are:

(1) Construction of the real numbers starting from scratch (that is from the set of counting numbers);
(2) Sequences and series of real numbers;
(3) Continuity of real valued functions of one real variable;
(4) Differentiability of real valued functions of one real variable;
(5) The Riemann integral of real valued functions of one real variable;

The listed topics form the “must have” list of topics. They look familiar from the Calculus I and II books. But in each of them, there will be some extra intriguing sub-topics such as algebraic numbers, unordered series (nets), deeper study of the points of discontinuity of monotone functions, as well as the points where a convex function might not be differentiable, examples of continuous and nowhere differentiable functions to mention just a few.

(6) If time permits, some basics of (general) topology (such as definition and first properties of topological spaces, metric spaces as most important for our purposes examples of topological spaces, compactness etc.). The basics meant here can by no means be a substitute for a more thorough learning of the beautiful and important subject Topology. They would merely introduce the right language for setting, and show the right perspective for treating some results in Calculus.

The topics above occupy chapters 1 through 6 (about 150 pages) in the book to be used in the course. The book itself looks small (compared to the usual Calculus books), but it actually contains more than one can find in any (standard) Calculus book. This is achieved by using very precise wording, best possible proofs, and meticulously well chosen examples for work both in-class and at-home.

A word of warning to the students: if a student wants to be successful in this class, they have to read the book very carefully,with attention to both general theory in the theorems and the details in the examples and remarks throughout the text. The Exercises sections, also much smaller than the (usual) Calculus books' ones, contain some more demanding examples which could be called Problems. In general, the suggested exercises are harder (some even significantly so) than the ones from Calculus I and II. This comes for good reason. As the author puts it (page ix, top), and the Instructor of the course agrees with him: at the level of Advanced Calculus, “more is learned by spending hours, if necessary, on a few problems, sometimes a single problem, than in routinely dispatching a dozen exercises, all following the same pattern. I hope the reader is not discouraged by difficulty, but rewarded by difficulty overcome”. To say it again, the computational aspects of Calculus are a prerequisite for this

course. Advanced Calculus is the place and the time for real Math. Be prepared for this!

There will be two Midterms and a Final Exam performed during the course.

There will also be several Quizzes and “turn-in” homeworks assigned.

All the problems for Exams will be taken from those suggested by the Instructor for work at home.

The overall grade will be based on 15% of the Turn-in homework's total score, 15% of the total Quizzes' score, 30% of the total of the Midterms' scores, and 40% of the Final Exam score. The scale for the overall grades follows:

Example: Suppose a student has A points total on the HW, B points total on the Quizzes, C points total on the Midterm Exams, and D points on the Final Exam. Suppose further that the maximal possible points one can get on these are A’, B’, C’, and D’ respectively. Then, one can compute a number S by the formula
$S = [15*A + 15*B + 30*C + 40*D] / [15*A' + 15*B' + 30*C' + 40*D']$.

The overall grade of the student above is determined now by the scale:

$0.92 < S$: A	$0.89 < S < 0.92$: A-	$0.86 < S < 0.89$: B+
$0.78 < S < 0.86$: B	$0.75 < S < 0.78$: B-	$0.71 < S < 0.75$: C+
$0.62 < S < 0.71$: C	$0.58 < S < 0.62$: C-	$0.55 < S < 0.58$: D+
$0.49 < S < 0.55$: D	$0.46 < S < 0.49$: D-	$S < 0.46$: F

No make-over exams will be scheduled.

Remark: The Instructor reserves the right to make any changes he considers academically advisable. Any such changes will be announced in advanced in class or by posting them to the e-mail accounts of the students. The students are responsible to be aware of the changes announced this way.