Syllabus for MAS 4316 Introduction to Commutative Algebra
Class Number 90436, Fall 2022

Book (main source): Algebraic Geometry and Commutative Algebra by S. Bosch

Course Description

Commutative Algebra (CA) is a branch of Mathematics whose objects of study are commutative rings (with identity) and modules over them. In this respect, CA is an extension of the Linear Algebra whose objects are fields (a special types of rings), and vector spaces (modules over a field). The main goal of CA is to classify, in a reasonable way, the commutative rings and the modules over them. This classification heavily depends on the rings themselves: the closer the rings to fields (in a certain sense), the simpler the classification of modules over them. The main motivation to study CA is its vast and powerful applications in different areas of mathematics, most notably Algebraic Number Theory and Algebraic Geometry.

In this course, we are introducing basic concepts of CA and are developing some elementary methods for studying these concepts. Numerous examples are provided in order to illuminate the theory developed.

Commutative rings are studied basically through their ideals. Finiteness conditions on ideals are used to distinguish special types of rings (Noetherian and Artinian). A rough classification of the Noetherian rings is provided by their Krull dimension. Primary factorization of ideals is proved as a generalization of the concept of unique presentation of elements of a ring as products of irreducible elements. The primary factorization is strengthened in the case of Dedekind domains. Integral extensions of rings are studied, existence of a normalization of a ring proved. Krull's Principal Ideal Theorem is proved as well.

Modules are studied. Categorical constructions involving modules and rings (such as localization, product, co-product, and tensor product) are introduced and used in the developed theory.

Course Objectives

The course carefully introduces the concepts of commutative rings and modules over them, as well as useful constructions on them, including products, co-products, quotient modules and rings, tensor product, and localization of rings and modules. The course thoroughly studies the properties of Noetherian rings and modules, including primary decomposition, integrality, Krull dimension, normalization of a commutative ring, and properties of Dedekind domains.

Course Learning Outcomes

Upon successful completion of this course, students should be able to demonstrate the following competencies:

• explain the fundamental concepts, constructions, and theorems related to Noetherian rings and modules, including Artinian rings and Dedekind domains;

• apply this knowledge to analyze particular rings, such as affine algebras, and modules over them;

• communicate eloquently, through an oral presentation, the material they have learned.

Prerequisites

A must have one: MAS 4301 (Algebraic Structures),

A Desirable one: MAS 4302 (Topics in Algebraic Structures)

Course Reading

1) Introduction to Commutative Algebra by Atiyah and Macdonalds;
2) Algebraic Geometry and Commutative Algebra by S. Bosch;
3) Introduction to Commutative Algebra and Algebraic Geometry by E. Kunz;
4) Commutative Algebra by H. Matsumura;
5) An Algebraic Introduction to Complex Projective Geometry I (Commutative Algebra) by Ch. Peskine;
6) Undergraduate Commutative Algebra by M. Reid;

Topics to be covered (subject to reasonable changes depending on the interests of the Instructor)

1. Rings, Ideals, and Maps
   (Notation and Examples, Homomorphisms and Isomorphisms, Ideals and Quotient Rings,
   Prime Ideals, Unique Factorization Domains)
2. Modules
   (Notation and Examples, Submodules and Maps, Tensor Products, Operations on Modules)
3. Localization
   (Notation and Examples, Ideals and Localization, UFD's and Localization)
4. Chain Conditions
   (Noetherian Rings, Noetherian Modules, Artinian Rings)
5. Primary Decomposition
   (Definitions and Examples, Primary Decomposition)
6. Integral Closure
   (Definitions and Notation, Going-Up, Normalization and Nullstellensatz, Going-Down,
   Examples)
7. Krull's Theorems and Dedekind Domains
   (Krull's Theorems, Dedekind Domains)

Monitoring of students' progress in class

The success of the students in class will be monitored based on completing Homework Assignments, two Midterm Tests, and a Final Exam.

Grading

The overall grade of the students will be determined as follows:

30% of the grade based on the Homework Assignments,

45% of the grade based on the Exams, and

25% of the grade based on the work of the students in class.

Academic Misconduct Statement

Florida International University is a community dedicated to generating and imparting knowledge through excellent teaching and research, the rigorous and respectful exchange of ideas and community service. All students should respect the right of others to have an equitable opportunity to learn and honestly to demonstrate the quality of their learning. Therefore, all students are expected to adhere to a standard of academic conduct, which demonstrates respect for themselves, their fellow students, and the educational mission of the University. All students are deemed by the University to understand that if they are found responsible for academic misconduct, they will be subject to the Academic Misconduct procedures and sanctions, as outlined in the Student Handbook.

Academic Misconduct includes: **Cheating** – The unauthorized use of books, notes, aids, electronic sources; or assistance from another person with respect to examinations, course assignments, field service reports, class recitations; or the unauthorized possession of examination papers or course materials, whether originally authorized or not. **Plagiarism** – The use and appropriation of another's work without any indication of the source and the representation of such work as the student's own. Any student who fails to give credit for ideas, expressions or materials taken from another source, including internet sources, is responsible for plagiarism.

To learn more about the academic integrity policies and procedures visit integrity.fiu.edu (Links to an external site.)

Accessibility and Accommodation

The Disability Resource Center collaborates with students, faculty, staff, and community members to create diverse learning environments that are usable, equitable, inclusive and sustainable. The DRC provides FIU students with disabilities the necessary support to successfully complete their education and participate in activities available to all students. If you have a diagnosed disability and plan to utilize academic accommodations, please contact the Center at 305-348-3532 or visit them at the Graham Center GC 190.

For additional assistance please contact FIU's Disability Resource Center

Important remark: The Instructor reserves the right to make any changes he considers academically advisable. Any such changes will be announced in advanced in class or by posting them to the e-mail accounts of the students. The students are responsible to be aware of the changes announced this way.