Title: Number Theory, MAS 4203, Section U01, Class Number 82688, Fall 2025

Lecture Notes: MAS 4203 Number Theory (provided by the Instructor)

Description of the course (syllabus):

The course presents an algebraic approach to elementary number theory. The motivating goal is to learn how to solve equations of one or more variables over the integers (a.k.a. Diophantine equations). The course begins by covering standard topics on divisibility of integers and some their applications (Chapters 1-5 in the current version of the Notes). These include the long division algorithm and the related concepts of greatest common divisor and least common multiple, solving linear Diophantine equations (of two and more unknowns), and the Fundamental Theorem of the Arithmetic of the rational integers. We discuss in detail the Pythagorean equation $X^2 + Y^2 = Z^2$. All considerations are done in the realm of the ring of integers Z.

Next, we introduce the ring Z/nZ of "integers modulo n", and develop its theory (a.k.a. Modular Arithmetic) as much as is convenient for the course (Chapter 6). The presentation is geared toward solving polynomial equations over this ring. Main results here are the Chinese remainder Theorem, the useful Fermat's Little Theorem, its generalization by Euler, and Wilson's Theorem. This naturally moves, in Chapter 7, to discussing a general technique for solving polynomial equations in Z/nZ. This technique is based on the very important Hensel's Lifting Lemma.

Chapter 8 contains a theorem which is a jewel in the theory of numbers: The Law of Quadratic Reciprocity of Legendre-Gauss. Gauss himself, who used to give names to the most important theorems he proved, called the theorem Golden Theorem (Theorema Aureum). It is related to finding the precise conditions when an equation of degree two has solutions in Z/nZ, but reveals a property of integers which has inspired much of the development in Algebraic Number Theory for the last more than 200 years.

If time permits, we will cover the theory of solving binomial equations, of any degree, in Z/nZ. This theory ultimately leads to revealing the structure of the group of invertible elements in that ring, and is associated with the concept of primitive roots modulo n.

Teaching Policy

After every section of the Notes covered in class, there will be a homework assignment posted on the web site of the course. Some of the problems from those assignments will be graded (there will be three "turn-in" assignments). The students' success in learning the material will also be assessed by one Midterm, and a Final Exam.

Grading policy

The overall grade of the students will be determined by taking

15% of the HW grades
30% of the Midterms' grade
35% of the Final Exam grade.

Example: Suppose a student has A points total on the HW, C points on the Midterm Exams, and D points on the Final Exam. Suppose further that the maximal possible points one can get on these are A', B', C', and D' respectively. Then, one computes a number S by the formula

$$S = [15*A + 30*C + 35*D] / [15*A' + 30*C' + 35*D']$$

The overall grade of the student above is determined now by the scale:

0.92 < S : A	0.89 < S < 0.92 : A-	0.86 < S < 0.89 : B+
0.78 < S < 0.86 : B	0.75 < S < 0.78 : B-	0.71 < S < 0.75 : C+
0.62 < S < 0.71 : C	0.58 < S < 0.62 : C-	0.55 < S < 0.58 : D+
0.49 < S < 0.55 : D	0.46 < S < 0.49 : D-	S < 0.46 : F

Make-up exams: No make-up exams will be given.

Note: The Instructor reserves the right to make changes in this Syllabus whenever academically acceptable.

Academic Misconduct Statement

Florida International University is a community dedicated to generating and imparting knowledge through excellent teaching and research, the rigorous and respectful exchange of ideas and community service. All students should respect the right of others to have an equitable opportunity to learn and honestly to demonstrate the quality of their learning. Therefore, all students are expected to adhere to a standard of academic conduct, which demonstrates respect for themselves, their fellow students, and the educational mission of the University. All students are deemed by the University to understand that if they are found responsible for academic misconduct, they will be subject to the Academic Misconduct procedures and sanctions, as outlined in the Student Handbook.

Academic Misconduct includes: **Cheating** – The unauthorized use of books, notes, aids, electronic sources; or assistance from another person with respect to examinations, course assignments, field service reports, class recitations; or the unauthorized possession of examination papers or course materials, whether originally authorized or not. **Plagiarism** – The use and appropriation of another's work without any indication of the source and the representation of such work as the student's own. Any student who fails to give credit for ideas, expressions or materials taken from another source, including internet sources, is responsible for plagiarism.

To learn more about the academic integrity policies and procedures visit integrity.fiu.edu (Links to an external site.)

Accessibility and Accommodation

The Disability Resource Center collaborates with students, faculty, staff, and community members to create diverse learning environments that are usable, equitable, inclusive and sustainable. The DRC provides FIU students with disabilities the necessary support to successfully complete their education and participate in activities available to all students. If you have a diagnosed disability and plan to utilize academic accommodations, please contact the Center at 305-348-3532 or visit them at the Graham Center GC 190.

Important remark: The Instructor reserves the right to make any changes he considers academically advisable. Any such changes will be announced in advanced in class or by posting them to the e-mail accounts of the students. The students are responsible to be aware of the changes announced this way.